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TRABAJO FIN DE GRADO EN

Economía

VACCINATION SCENARIOS FOR THE COVID-19 PANDEMIC IN SPAIN

Módulo:

Análisis Económico

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ABSTRACT

This paper simulates the evolution of the COVID-19 pandemic in Spain. The attention is focused on providing a guidance to vaccination policy. The quantitative results show the importance of having a high efficacy and administering speed. Herd immunity can be achieved by the end of summer 2021 with a correct design of the vaccination programme and this paper gives prove of this.

The analysis goes from the 28th of January 2020 up to the 29th of January 2022 and it gives accurate numerical information on the daily and total number of infected, susceptible, recovered, dead and vaccinated individual.

RESUMEN

Este trabajo simula la evolución de la pandemia COVI-19 en España. El principal foco de atención se centra en orientar la política de vacunación. Los resultados son cuantificados y muestran la importancia de tener una alta eficacia y ritmo de administración de la vacuna. La inmunidad del rebaño se puede alcanzar para finales del verano 2021 con un correcto diseño del programa de vacunación y este trabajo es la prueba de esto mismo.

El análisis comienza el 29 de enero del 2020 y acaba el 28 de enero del 2022 dando información precisa del número tanto diario como total de personas infectadas, susceptibles, recuperadas, muertas y vacunadas.

Keywords: Herd immunity timing, efficacy rate, immunity rate, vaccination speed and recovery estimation.

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INTRODUCTION

Ever since the outbreak of the pandemic cause by the virus SARS-CoV-2 (COVID-19), every country around the world has summed forces to fight it back as soon as possible. The pandemic was declared by the WHO (World Health Organization) on the 11th of March of 2020, just a bit more than two months after the first infected individual was detected on the 31st of December 2019 in Wuhan (China). Around one year after this, the vaccine to the virus finally arrived on the 8th of December 2020 when a 90-year-old British woman became the first world vaccinated individual with one dose for the COVID-19 (BBC News, 2020).

As for April 2021, he vaccine for the virus SARS-CoV-2 or commonly known COVID-19 has become the centre of attention for every country, institution, and human being around the globe. The fast spread of the virus has triggered utterly devastating sanitarian and economic consequences. In fact, as for the 31st of December 2020 there were 1,803,334 deaths around the world and for the 10th of March 2021, the last day for which I will consider official data, the number of total world deaths were 2,608,898 people. Consequently, carrying out a proper vaccination policy has gained great importance. Many countries have devoted enormous amounts of resources to the production and commercialization of the vaccine as this one happens to be the fastest way out of this emergency.

For the case of Spain, the first COVID-19 infected individual arrived on the 28th of January 2020. Forty-six days after this, the government declared the SoA (State of Alarm) on the 14th of March 2020, which is actually three days after the WHO (World Health Organization) had declared the situation as a pandemic. Along 2020 (the first year of the pandemic) when there was yet no vaccination plan, numerous restrictions were implemented to Spanish citizens trying to control the spread of the virus: lockdown (for more than 3 months), closure of businesses, institutions, public places, social and physical distancing, self-isolation, travel restrictions, curfew… and so on.

Ever since the vaccine was introduced in Spain the number of new cases along with daily deaths have been little by little decreasing and the situation has got under control. The first vaccinated individual with two doses arrived on the 29th of January 2020, Still, as for the 10th of March 2021 only 1,6247 million people have been vaccinated in Spain, which only represents 3.46% of the whole Spanish population.

The aim of this paper is to analyse the evolution of the pandemic in Spain for two years (730 days) after the first individual for COVID-19 was detected on the 29th of January 2020 (day one in the model). The motivation behind is to estimate the exact time when the country will reach the so claimed herd immunity and to analysis how this timing may vary according to different variables such as the efficacy rate of the vaccine or the vaccination speed. To do so, the paper implements a SIR-type model (compartmental model for modelling infectious diseases) in MATLAB which among other things analysis data and estimates daily number of infected, susceptible, death, recovered or vaccinated individuals (Foppa, 2017). The results of the paper may be useful for vaccination policy makers to assess current weaknesses regarding vaccination speed and for them to be able to lead Spain into a vaccination path that truly reaches herd immunity by the end of summer 2021 which happens to be the aim of the Spanish government.

In the first section of the paper the model is described in which is comprised of fourteen equations. In the second section of the paper this model is calibrated to represent the evolution of the virus COVID-19 for the case of Spain. In the third section of the paper, I am carrying out the analysis. First, I am presenting the results for the calibrated baseline model and later on, I am analysing two different what if scenarios in which I am changing the calibrated values under the baseline model of the efficacy rate and of the vaccination speed.

MODEL DESCRIPTION

For the first day in the model I am about to present total population is going to be exogenous, and it is allocated into five different groups of people regarding the health state: susceptible (N_t^S), infected (N_t^i), recovered (N_t^r), dead (N_t^d), and vaccinated & immune (N_t^{sv}) individuals.

$$N = N_t^S + N_t^i + N_t^r + N_t^d + N_t^{sv} \qquad (1)$$

Where,

- **Susceptible individuals** are healthy people in day t that given the presence of the virus COVID-19 present a likelihood of becoming infected.
- **Infected individuals** are people that are infected in day t by the virus and can transmit it to other people. These individuals might be either in the incubation period or in the outcome phase as it will be later presented.
- **Recovered individuals** are people that got infected in the past and that have got over the virus in day t. These people cannot be susceptible individuals anymore as I will assume that they have developed antibodies for the COVID-19. Thus, they are all considered to be immune to the virus and will behave as vaccinated individuals.
- **Dead individuals** are people that got infected by the virus in the past and have not been able to recover. These are the number of deaths for any given day t.
- **Vaccinated & immune individuals** are the number of individuals that have been effectively vaccinated and have turned immune to the virus COVID-19 as for day t.

The first day on the pandemic (day 1, $t = 1$) corresponds to the arrival of the first infected person, in which, $N_t^i = 1$, $N_t^r = N_t^d = N_t^{sv} = 0$ and $N_t^S = N - 1$. Regarding the vaccination period it will start the 28th of January 2021 which represents day $t = 365$ in our model (one exact year after the first infected individual is detected). Therefore, at the beginning of the model, the number of vaccinated individuals (N_t^v) and the number of immune individuals after having been vaccinated (N_t^{sv}) will be zero and are no relevant in the model.

On any given day, individuals go from being susceptible to infected by "new daily cases" function (equation 2). After they get infected, they can either survive with probability $(1-\lambda_t)$ through the "recovered" function (equation 10) or they may die with probability λ_t through the "deaths" function (equation 11). Eventually, the vaccine will arrive and every day, a proportion of the population will be vaccinated through equation 12. However, only a fraction $(1-\theta)$ of the total number of vaccinated individuals will turn out to be immune (equation 13). This fraction will be immune to the virus and will no longer be treated as susceptible individuals (equation 14).

In total, the model is explained by the following 14 equations that estimate the evolution of the pandemic caused by the virus COVID-19.

$$N = N_t^S + N_t^i + N_t^r + N_t^d + N_t^{sv} \quad (1)$$

$$N_t^n = \alpha_t y_t q_{t-1}\left(\frac{N_{t-1}^i}{N-N_{t-1}^d}\right) N_{t-1}^{ss} \quad (2)$$

$$q_t = \left[1\frac{N_t^{inc}}{N_t^i} + \delta\frac{N_t^{out}}{N_t^i}\right](1-v_t) \quad (3)$$

$$v_t = v_0\left(1 - v_1 \sum_{i=1}^{7} \frac{N_{t-i}^{ik}}{7}\right) \quad (4)$$

$$y_t = y_0 - \eta\left(\frac{\sum_{j=1}^{7} N_{t-j}^{ik}}{7}\right) \quad (5)$$

$$N_t^{ik} = (1-q_t)N_t^i \quad (6)$$

$$N_t^i = N_{t-1}^i + N_t^n - \frac{1}{T_P}\sum_{j=T_i}^{(T_i+T_p)-1} N_{t-j}^n \quad (7)$$

$$N_t^{inc} = \sum_{j=0}^{T_i-1} N_{t-j}^n \quad (8)$$

$$N_t^{out} = \sum_{j=0}^{(T_p-1)} \frac{(T_p-j-1)}{T_p} N_{t-T_i-j}^n \quad (9)$$

$$N_t^r = N_{t-1}^r + (1-\lambda_t)\frac{1}{T_p}\sum_{j:T_i}^{(T_i+T_p)-1} N_{t-j}^n \qquad (10)$$

$$N_t^d = N_{t-1}^d + (\lambda_t)\,\frac{1}{T_p}\sum_{j:T_i}^{(T_i+T_p)-1} N_{t-j}^n \qquad (11)$$

$$N_t^v = N_{t-1}^v + \varphi N \qquad (12)$$

$$N_t^{sv} = \theta(N_{t-1}^{sv} + \varphi N) \qquad (13)$$

$$N_t^{ss} = N_t^s - N_t^{sv} \qquad (14)$$

The different exogenous and endogenous variables determining the model from equation 2 up to 13 listed above will be now explained one by one.

New cases function

In any given day t, new cases depend on five elements: the contagion probability (α_t), the number of daily contacts (y_t) , the quarantine factor for those infected individuals not detected (q_{t-1}), the infectious rate $\left(\frac{N_{t-1}^i}{N-N_{t-1}^d}\right)$ and the number of susceptible individuals until the day before (N_{t-1}^{ss}). The bigger any of these 5 elements is the greater the number of new cases will be as follows:

$$N_t^n = \alpha_t y_t q_{t-1}\left(\frac{N_{t-1}^i}{N-N_{t-1}^d}\right) N_{t-1}^{ss} \qquad (2)$$

Notice that $\alpha_t y_t q_{t-1}\left(\frac{N_{t-1}^i}{N-N_{t-1}^d}\right)$ is the infection probability for one susceptible individual and it is multiplying the number of susceptible individuals as for the day before (N_{t-1}^{ss}) who have either not been vaccinated or have been vaccinated but are not immune to the current strength of the virus (once the vaccine is implemented). Therefore, those individuals who have been effectively vaccinated and have become immune to the virus are subtracted (N_t^{sv}) from those sill susceptible individuals on any given day t (N_t^S) as it is represented by equation number 13:

$$N_t^{SS} = N_t^S - N_t^{sv} \qquad (14)$$

Next, I am going to explain the elements that are displayed in equation (2).

Contagion probability

Firstly, the contagion probability (α_t) is the probability of the transmission of the virus because of the contact of a non-infected person with an infected one which is unique to the type of virus in the analysis. This variable is exogenous in the model and is based on updated information on how contagious the virus is. Such transmission will be changing as new information about the virus COVID-19 is known, as different preventive measures take place (wearing face masks, social distance among others), with the seasonal spread of the virus… and so on.

Quarantine factor

Secondly, the quarantine factor (q_t) is a ratio that measures the fraction of those individuals who are infected but are not following a quarantine and have the potential to transmit the virus to susceptible individuals. Thus, it is a ratio that can take values between zero and one. This factor is endogenous in the model and it depends on other endogenous and exogenous variables:

$$q_t = \left[1\frac{N_t^{inc}}{N_t^i} + \delta\frac{N_t^{out}}{N_t^i}\right](1 - v_t) \qquad (3)$$

The quarantine factor depends on the rate of asymptomatic individuals in each of the two phases (once an individual has been infected) because asymptomatic individuals transmit the virus without been able to be detected and are key determinants for new cases. The two phases are the incubation period and the outcome phase. After the incubation period, infected individuals enter the outcome phase, but they do not necessarily begin to develop symptoms. In fact, the fraction of those in the outcome phase not developing symptoms (asymptomatic) and having the potential to transmit the virus without been detected is captured by delta (δ) in the model. In general, the more asymptomatic individuals there are, the greater the quarantine factor will be and the greater the number of potential new cases will be.

Consequently, the quarantine factor is dependent on infected individuals in each time $t\ (N_t^i)$. However, those individuals that get the virus might be in either in the incubation period ((N_t^{inc}) still developing the virus) which will last (T^i) days), or in the outcome phase (N_t^{out}) which will last (T^p) days. On average, a person that gets infected will spend around $\left(T^i + \frac{(T^p+1)}{2}\right)$ days infected until recover or death.

There is another determinant in the quarantine factor named as the testing rate (v_t). Public health department plays an especially important role detecting infected individuals by carrying out COVID-19 tests to citizens. The testing rate in the model considers those tests carried out on infected individuals that not having symptoms for the virus happened to be tested most probably because they have currently been in contact with just infected individuals. Therefore, the testing rate (v_t) is introduced in equation (3) to determine the asymptomatic cases that cannot be detected through diagnosis testing $(1 - v_t)$.

The testing rate has its own equation in the model which depends on the number of infected known individuals. The more infected known individuals there are, the lower the testing rate would be as the testing system would saturate and the capacity to detect infected individuals running tests will decrease. Consequently, at most, the testing rate can be v_0 and depending on the daily average (with past week data) of infected known individuals it will be decreased. Besides, v_1 is an exogenous fixed parameter introduced for the equation to give a reasonable number. The testing rate is therefore explained by the following equation:

$$v_t = v_0 \left(1 - v_1 \sum_{i=1}^{7} \frac{N_{t-i}^{ik}}{7}\right) \qquad (4)$$

In conclusion, the quarantine factor is a ratio that shows the fraction of those infected individuals that are transmitting the virus because they are not isolating themselves from the rest of the population (they are not following the quarantine), a population in which we find susceptible people that can get infected by the virus (susceptible individuals). The higher the quarantine factor, the higher the number of people that are infected and not detected which implies that the higher the number of potential new cases will be. Thus, the ratio can be interpreted as a multiplier of the infection rate for any given day t (observe equation 2).

Nevertheless, the reasons for an individual not to follow a quarantine can be two: those infected might be asymptomatic and might not know they have the virus or those that have symptoms might be acting in negligence not reporting it to local authorities. As far as the model is concerned, I am going to presume that we are under a socially responsible society in which those acting in negligence are a minority and can be disregarded. Anyway, these asymptomatic people happen to be a threat for those healthy susceptible individuals and are key determining new daily cases.

In other words, this ratio shows how effective a certain society is in detecting or capturing those infected individuals that have the potential to transmit the virus. The main problem is that the higher the number of infected individuals is, the more difficult it would be for local authorities to detect these individuals and isolate them (higher quarantine factor).

Number of interpersonal contacts

Thirdly, the number of interpersonal contacts per individua depends inversely on the virus incidence but it is controlled by the government through social distancing restrictions to contain the virus spread as follows:

$$y_t = y_0 - \eta\left(\frac{\sum_{j=1}^{7} N_{t-j}^{ik}}{7}\right) \qquad (5)$$

$$N_t^{ik} = (1 - q_t)N_t^i \qquad (6)$$

In equation (5), η determines the policy coefficient which measures how aggressive the policy carried out by the government is towards wiping out the contagion effect of the virus COVID-19. This coefficient multiplies the daily average number of known infected individuals $\left(N_t^{ik}\right)$ counting data over the past week (seven days average). Thus, the number of known infected individuals, as it is shown in (6), is the fraction of those individuals who have been detected thanks to the COVID-19 tests or the presence of symptoms (in which case we are assuming that they do follow a quarantine by isolating themselves from the rest of the population).

Vaccinated individuals

At some point in the analysis of the evolution of the pandemic, the vaccine to the virus COVID-19 is going to arrive. I am going to consider that, from that moment onwards and regardless of the situation, a fixed proportion of the population represented by the Greek letter φ is going to be vaccinated every single day. The fixed proportion will be addressed in the paper as the daily rate of vaccine administration (φ). Consequently, on any given day, the number of vaccinated people will be those individuals vaccinated up to the day before (N_{t-1}^v) plus a fixed proportion of individuals over the fixed population vaccinated that day (φN).

$$N_t^v = N_{t-1}^v + \varphi N \qquad (12)$$

However, given the prompt development of the vaccine for the virus, the efficacy rate (the proportion of vaccinated individuals that do not turn immune after vaccination because of not having a completely effective vaccine) of the different developed vaccines is close but lower than 100%. This fact will affect the immunity rate (θ) in the model, which is the daily fraction of the population that is vaccinated and immune to the virus. Thus, every day, a fixed proportion of those vaccinated individuals will not turn immune to the virus $(1-\theta)$ and will still be treated as susceptible individuals. Notice that the immunity rate is a cumulative rate that daily applies to every vaccinated individual, regardless of whether they were vaccinated someday in the past (N_{t-1}^{sv}) or that very same day (φN). Because it is cumulative, as time goes by, the probability of not becoming immune to the virus even if you have been vaccinated will be greater. This has also to do with the threat that the virus COVID-19 may mutate, and the current vaccine might turn out ineffective on the new mutation. All this is captured in the following equation which gives the number of individuals that were susceptible but thanks to having been vaccinated become immune to the virus (N_t^{sv}) .

$$N_t^{sv} = \theta(N_{t-1}^{sv} + \varphi N) \qquad (13)$$

Infected individuals

In any given day t, the number of infected individuals (active cases) is going to be obtained as the result of three effects: the number of infected individuals the day before (N_{t-1}^{i}), the number of new cases for that day t (N_t^n) and the number of individuals who cease to be infected $\left(\frac{1}{T_P}\sum_{j=T_i}^{(T_i+T_p)-1} N_{t-j}^{n}\right)$. The greater the number of infected individuals the day before or the number of new cases for the given day t, the greater the number of infected individuals in t. However, the greater the number of individuals who are no longer infected because they either recover from the virus or because they die, the lower the number of infected individuals in day t.

$$N_t^i = N_{t-1}^i + N_t^n - \frac{1}{T_P}\sum_{j=T_i}^{(T_i+T_p)-1} N_{t-j}^{n} \qquad (7)$$

The third term in the right-hand side of the equation (7) calculates the number of individuals who are no longer infected as a fix proportion $\frac{1}{T_P}$ (given that T_P is the number of days in the outcome phase), over the sum of all new cases that happened in the past for which individuals are still in the outcome phase of the illness.

Individuals in the incubation period

As it has been already explained, infected individuals can either be in the incubation period which accounts for the first T^i of the illness or in the outcome phase which follows the incubation period up until at most T^p days.

For any given day t, the number of people in the incubation period is the sum of all individuals that got infected at most $T^i - 1$ days ago considering that new cases of infected individuals in day t belong to those in the incubation period.

$$N_t^{inc} = \sum_{j=0}^{T_i-1} N_{t-j}^n \qquad (8)$$

Individuals in the outcome phase

Following the same rational, at any given day t, the number of individuals infected and belonging to the outcome phase are the sum of those infected individuals that got the virus COVID-19 at least T_i days ago, until at most $(T_i + T_p) - 1$ days ago.

$$N_t^{out} = \sum_{j=0}^{(T_p-1)} \frac{(T_p-j-1)}{T_p} N_{t-T_i-j}^n \qquad (9)$$

The fraction $\left(\frac{(T_p-j-1)}{T_p}\right)$ represents those individuals that remain in the outcome phase which implies that $1 - \left(\frac{(T_p-j-1)}{T_p}\right)$ leave the outcome phase because they either survive or die. For instance, when $j = 0$, the model is considering only those individuals in the first day of the outcome phase and the model shows that a proportion of $\frac{T_p - 0 - 1}{T_p}$ individuals remain in the outcome phase while $\frac{1}{T_p}$ have exited it.

Recovered individuals

At any day t, the number of recovered individuals that survive from the COVID-19 is determined by the number of recovered individuals up to the day before (N_{t-1}^{r}) plus the proportion of those individuals that exit the illness because they get over it and do not die $\left((1-\lambda_t)\frac{1}{T_p}\sum_{j:T_i}^{(T_i+T_p)-1} N_{t-j}^{n}\right)$. Thus, it is particularly important to define the fatality rate of the virus (λ_t) which will determine the proportion of those individuals in the outcome phase that die (λ_t) or that recover $(1-\lambda_t)$. This ration will be exogenous in the model. Recall, that the right-hand side of the equation $\left(\frac{1}{T_p}\sum_{j:T_i}^{(T_i+T_p)-1} N_{t-j}^{n}\right)$ has already been explained in the previous section.

$$N_t^{r} = N_{t-1}^{r} + (1-\lambda_t)\frac{1}{T_p}\sum_{j:T_i}^{(T_i+T_p)-1} N_{t-j}^{n} \qquad (10)$$

Death individuals

Following the same aggregation scheme as for recovered individuals, dead individuals in any given day t are going to be the accumulation of those dead individuals up to the day before (N_{t-1}^{d}) and the proportion of those individuals that exit the outcome phase of the virus because they unfortunately die $\left((\lambda_t)\ \frac{1}{T_p}\sum_{j:T_i}^{(T_i+T_p)-1} N_{t-j}^{n}\right)$ where λ_t is the infection fatality rate.

$$N_t^{d} = N_{t-1}^{d} + (\lambda_t)\ \frac{1}{T_p}\sum_{j:T_i}^{(T_i+T_p)-1} N_{t-j}^{n} \qquad (11)$$

MODEL CALIBRATION FOR SPAIN

The model that has just been presented is going to be calibrated for the case of Spain. This is aiming to estimate how does the implementation of a vaccine alter the model in a medium-size country like Spain, and when will this country attain the so-claimed herd immunity. Experts believe that a country needs 60% to 70% of its population to be vaccinated for this to attain herd immunity. I am going to set herd immunity at 70% for the analysis section of the paper.

In conclusion, the goal of the paper is to determine under what circumstances and when will the desired herd immunity take place. In fact, nowadays, herd immunity happens to be the goal for every country around the world because it is believed to be the moment from which the chain of transmission of the virus is broken, most of the individuals are immune to the virus and the situation gets under control. No need to say that this is something countries are eager to attain these days given the economic and social consequences of the pandemic. (Dr Soumya Swaminathan, WHO´s Chief Scientist, 2020)

First of all, the first parameter to be calibrated is total population for the case of Spain which is going to be exogenous in the model and set at $N = 47$ million. This value is close to the real population in Spain at the beginning of 2020. (Eurostat, s. f.-b)

As far as the fatality rate of the virus COVID-19 is concerned, there are two different indicators that estimate the fatality rate: infection fatality rate (IFR), which is the number of confirmed deaths over confirmed and unconfirmed cases $\frac{confirmed\ deaths}{confirmed+unconfirmed\ cases}$, and the case fatality rate (CFR), which is the number of confirmed deaths over only confirmed cases $\frac{confirmed\ deaths}{confirmed\ cases}$. Thus, both indicators are equal as long as there exist no asymptomatic cases for the illness of analysis (no unconfirmed cases). However, it is well-known that a great proportion of infected individuals are asymptomatic at least for the case of Spain, which is translated as a lower IFR rate compared to CFR rate $(CFR > IFR)$. This fact can be observed in *table 1* below. As far as the model in this paper is concerned, it is going to measures IFR as it considers both asymptomatic and symptomatic cases.

Regarding data published by the Health department of Spain that can be observed in the following table, the estimations updated for the 21st of January of 2021 suggests a IFR of 0,8% while the CFR using only reported cases and deaths is set on 8%. Therefore, the number of asymptomatic cases is quite relevant in the estimation of new cases. According to the Health department of Spain, after the 11.7 days since an individual gets infected, 95% of them develop symptoms. However, during the incubation period, less than 20% of individuals develop symptoms. The model in this paper assumes that 100% of individuals in the incubation period (first 5 days of the illness) are asymptomatic and that 40% of individuals in the outcome phase (following days up to at most 25 days) are also asymptomatic. Consequently, the rate of asymptomatic cases in each period of the illness assumed in the model seems rational according to other studies and also after having observed the huge difference between CFR and IFR in *table 1*. (Gobierno de España, 2021)

Table 1 COVID-19 CFR and IFR in Spain by age ranges.

Observed CFR for COVID-19 in Spain and estimation of the IFR					
Age range (years old)	**Notified cases**	**Notified deaths**	**Observed CFR**	**Estimated cases**	**Estimated IFR**
Below 10	871	2	**0.23%**	110,406	**0.002%**
10-19	1,619	5	**0.31%**	185,416	**0.003%**
20-49	13,439	23	**0.17%**	926,676	**0.002%**
50-69	57,818	263	**0.45%**	724,151	**004%**
Above 70	88,094	16,559	**19%**	403,548	**4.1%**
TOTAL	**239,095**	**19,155**	8%	**2,350,198**	0.8%

Source: Own elaboration with data taken from the Ministry of Health of the Spanish government.

Nevertheless, CFR has not always taken the same value. At the beginning of the pandemic caused by the COVID-19, the case fatality rate (CFR) for the virus was believed to be around 3,4% according estimates reported in China (Anderson et al., 2020). However, as more estimates were carried out, it was said that around 40-81% of the population could be infected by the virus (out of which most of them may not develop symptoms). Due to this, the WHO (World Health Organization) has established the IFR around 1% and 0.9%. However, the IFR in the model for the case of Spain is going to be set at $\lambda_t = 0.008$ (0.80%). (Anderson et al., 2020)

The reasoning for setting a high IFR for Spain is that the virus is more fatal on the elderly and the elderly takes up a great share of the population in Spain. It has already been observed in *table 1* with data taken from the Ministry of Health of Spain that the IFR for those aged above 70 years old is 4.1% (way above average of 0.8%). In the next *table 2* it can be observed the risk of COVID-19 deaths by age group elaborated with data taken from CDC (Centers for Disease Prevention and Control) taking as reference probability of death for those aged between 15 and 17 years-old. For those individuals aged between 85 and 100 years-old the probability of dying is 8,700 times greater than for those individuals aged between 5 and 17 years-old. (CDC, 2021)

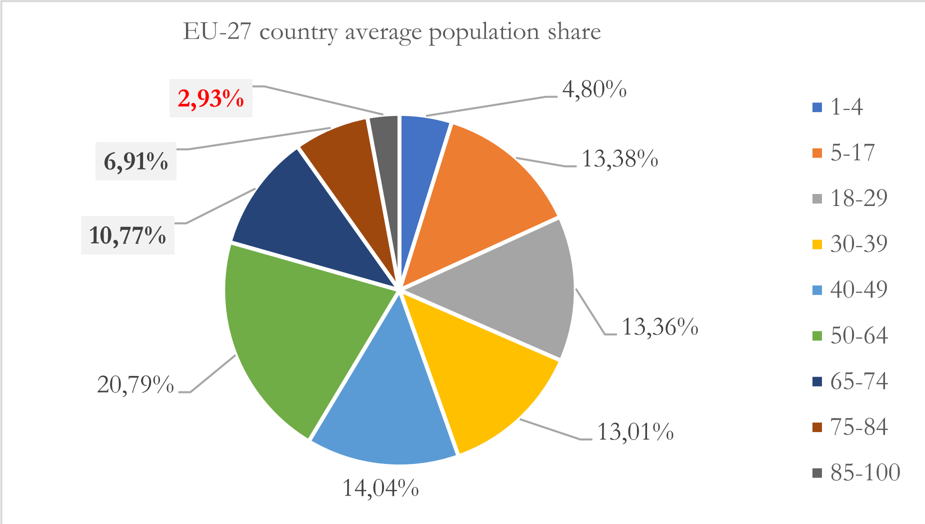
Table 2 Risk of dying from COVID-19 by age group.

Risk of dying from COVID-19									
Rate compared to 5-17 years old	**0-4** years old	**5-17** years old	**18-29** years old	**30-39** years old	**40-49** years old	**50-64** years old	**65-74** years old	**75-84** years old	**85-100** years old
Deaths	1x	*Reference group*	10x	45x	130x	440x	1300x	3200x	**8700x**

Source: Own elaboration with data taken from the Ministry of Health of Spain.

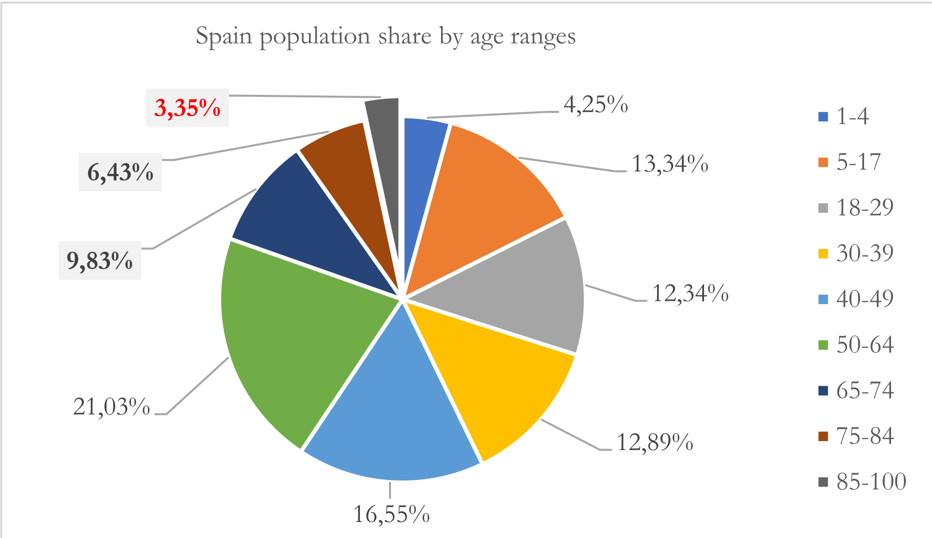
Once that is clear that the fatality rate skyrockets for those individuals with 85-100 years-old, it is time to compare the population share of this age group for the EU average and for Spain. In *figure 1* I have computed the share of each age range for an average EU country and by comparing it with *figure 2* in which we find age population shares by age ranges for Spain, it is observable that there are more individuals in this age group in Spain (3.35% > 2.93%). In fact, the difference takes up a 0.42% weight which considering the Spanish population it implies 200,000 more people in Spain with an extremely high risk of dying for COVID-19 according to *table 2*. This explains why the CFR is so high in comparison with other countries in Spain. Even if those aged between 65-74 and 75-84 years old take a greater share for an EU average, I am going to consider that the high probability of death of this last age group must be taken into account for the calibration of IFR parameter in the model. Consequently, the IFR in the model will be set at 0.80% (Anderson et al., 2020, Ioannidisa, 2021, Eurostat, 2021 and Català et al., 2021).

Figure 2 EU-27 average age group by population share.



Source: Own elaboration with data taken from Eurostat.

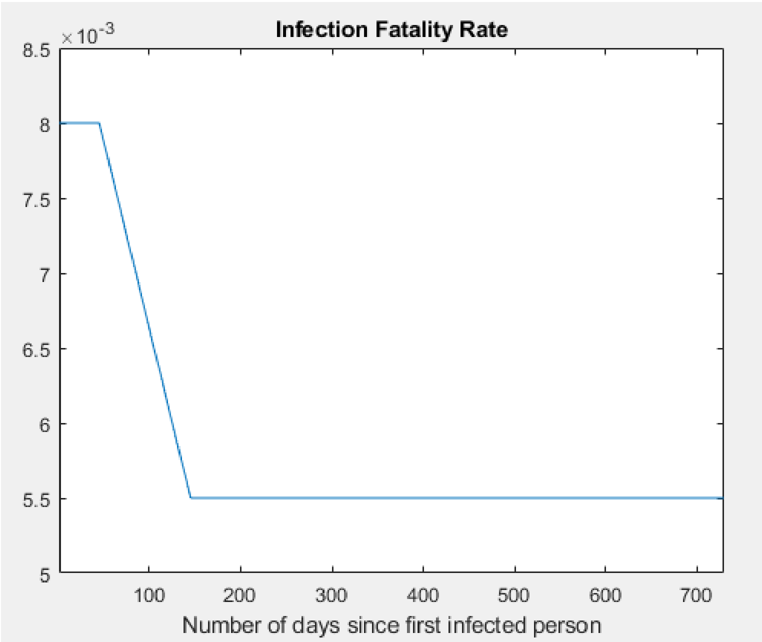
Figure 1 Spanish age group by population share.



Source: Own elaboration with data taken from Eurostat.

Once we have calibrated the initial value for the IFR in the model it is time to explain the evolution of this one over time. Following the State of Alarm declaration, the lockdown is stablished in Spain on the 15th of March of 2020 (day 46 since the first infected person in the model), the IFR will be altered to better show the reality of the situation, mainly because thanks to the lockdown, the uncontrolled spread of the virus will be hold back. Therefore, the infection fatality rate will experience a constant decrease (explained by the decrease in the number of confirmed deaths implicit in IFR definition) up to the end of the lockdown the 21st of June (day 144 in the model) reaching a value of 0.003%. Once the lockdown is over, sanitarian measures and social distancing are going to make it possible for the IFR to be unaltered and remain at 0.3% from that moment onwards up to the end of the analysis. (observe *figure 3*).

Figure 3 Infection Fatality Rate in the calibrated model



Source: Own elaboration.

Another issue to determine is the length of the incubation period and the outcome phase of the virus COVID-19. Regarding the incubation period, according to Anderson paper and the WHO reports, on average it is believed to last between 5 and 6 days. In the model, the incubation period will be established at $T_i = 5$ days. Besides, according to the Ministry of Health of Spain, an infected individual weakly affected by the virus is infected on average for 2 weeks, and an individual strongly affected (requiring hospitalization) by the virus can be infected between 3-6 weeks. However, the number of individuals that require hospitalization have been estimated to be 4,4% in Great Britain using cases obtained from China. Consequently, in the model, the average number of days an individual remains infected by the virus is going to be set in 18 days, closer to the length when an individual is weakly affected by the virus. Thus, on average the virus will last $T_S = 18$ days $\left(T_i + \frac{T_p+1}{2}\right)$ which depending on the specific health situation of the individual can last up to 30 days. As a result, length of the outcome phase is set at $T_p = 25$ days (Eubank et al., 2020).

In relation with the incubation and the outcome phase, as far as the proportion of asymptomatic individuals is concerned, the model assumes that 100% of those individuals that got infected and happen to be in the incubation period (first five days in the model) are asymptomatic. This is not too far from the estimations carried out by the Spanish Government that conclude that less than 20% of them develop symptoms in the first period of the illness. Besides, the fraction of those in the outcome phase not developing symptoms who have the potential to transmit the virus without been detected is captured by delta parameter (δ) in the model and it is going to be set in 40%. This means that 60% of the individuals in the outcome phase develop symptoms and isolate themselves while 40% of them are asymptomatic and keep transmitting the virus without knowing it.

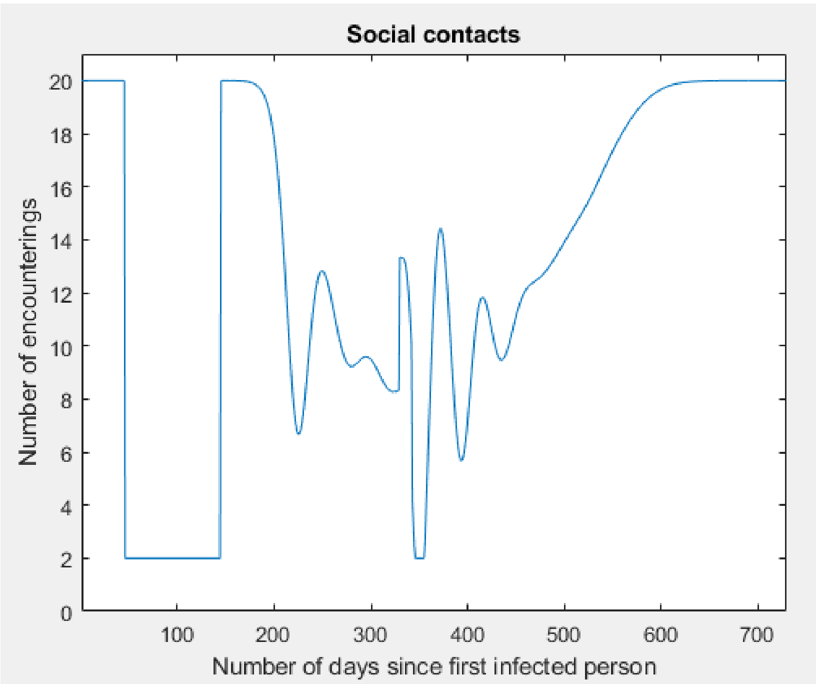
In conclusion, 40% rate of asymptomatic cases over an average of 18 days in the outcome phase accounts for 7.2 days without symptoms during the outcome phase. If I sum this to the first 5 days in the outcome phase in which 100% of them are asymptomatic cases, I get an overall average of 12.2 days with 100% of asymptomatic individuals in the model. Again, this is quite sensible given that after day 11.7 nearly 95% of them develop symptoms according to Spanish Government data (Gobierno de España, 2021).

Apart from this, the daily number of social interpersonal contacts is clearly country-specific as it depends on social culture and norms, weather conditions, working activity, type of job, amount of free time… and so on. Besides, this is variable is uncertain and it will vary in the model. As far as Spain is concerned, it is widely believed to be a country in which individuals gather quite often with friends, family, work mates... and so on. In the model, this value will be exogenous since day zero and under normal circumstances, before the state of alarm SoA which is stablished a day before the lockdown (the 14th of March), it will be set at $y_t = 20$. This is believed to correctly represent the real situation of Spain under normal circumstances when there is no SoA due to the threat of a virus.

Nevertheless, after SoA, y_t will be given by equation five in which $y_t = y_0 - \eta\left(\frac{\sum_{j=1}^{7} N_{t-j}^{ik}}{7}\right)$. For instance, once the lockdown is stablished, the number of interpersonal contacts will be decreased in 90% up to $y_t = 2$ social contacts per day (which represents average contacts between individuals belonging to the same family unit). Given the average number of children in Spain below 1.3, the number of 2 interpersonal contacts on average seems a reasonable estimation to consider in the model. Once the lockdown is over in day 144, the situation goes back to pre-lockdown values when $y_t = 20$ as it can be observed in *figure 4* (Casares & Khan, 2020 and INE, 2019).

As time goes by and depending on the previous week average of known daily infected individuals for any day t (equation number 5), the number of the maximum allowed interpersonal contacts will vary regarding governmental decisions trying to hold back the spread of the virus through the control of interpersonal contacts. According to the model, it can be observed how during the second wave of the virus (between days around 260-325 in the model) restrictive policies regarding the number of interpersonal allowed contacts were adopted. Besides, when Christmas time arrived, Spanish government allowed social contacts between at most 6 individuals which is represented in day 330 of the model (the 24th of December) by an increase in 5 the number of interpersonal contacts. Nevertheless, this situation will only last 13 days until the 6th of January of 2021 (day 343 in the model) when even more sever measures than before Christmas time are implemented (further decrease in social contacts) due to the rise of infected individuals during Christmas time.

Figure 4 Social contacts in the calibrated model.

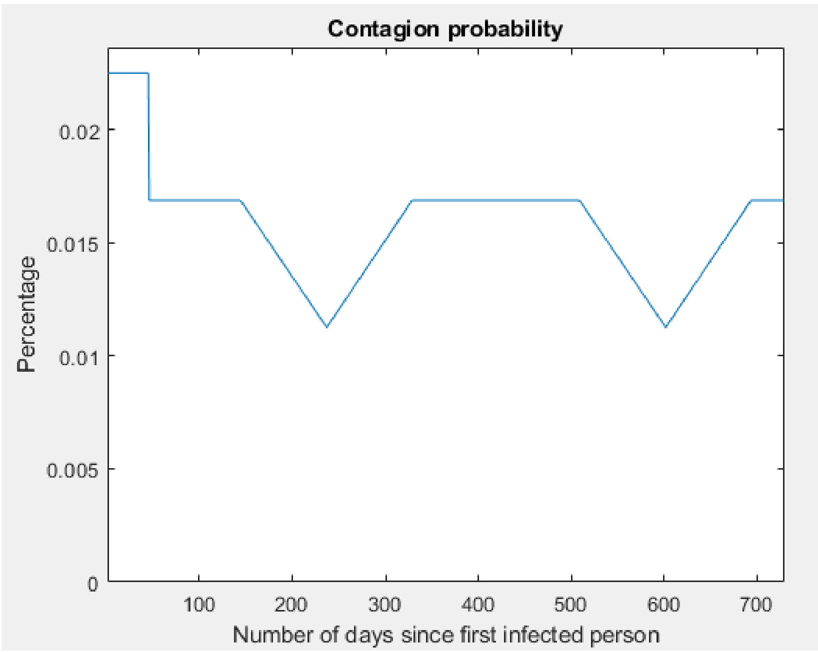


Source: Own elaboration.

Consequently, the more new cases there are, the lower the number of social contacts will be as a government mechanism trying to contain the situation before it goes out of hands. This has been captured by (η) in equation five which depends on daily average of known cases (taking data for the past week). However, once the vaccine is implemented (day 365 for the first vaccinated individual), control over social contacts to revert the situation won´t be that necessary as day after day more individuals will become immune to the virus. Therefore, after day 365 social contacts will only suffer to more waves (wave four and wave five) for which the second one will be less restrictive and little by little social contacts will increase up to initial levels around day 600 (20th of September 2021).

Another key factor in determining the number of daily new cases is the contagion probability α_t stablished in equation number two. This factor is exogenous in the model but changes along with different time dependent circumstances such as COVID-19 preventive measures (wearing a mask, keeping social distance, controlling capacity in locals... and so on) or with weather conditions that alter how people interact with each other. In the next *figure 5* it can be observed how the contagion probability is first set at a value of 0.0225 (2.25%) when individuals were interacting under normal circumstances without knowing about the threat of the COVID-19. In the paperwork carried out by Casares and Khan (2020) it is explained how they measured the initial value of the contagion probability for the case of

Figure 5 Contagion probability in the calibrated model.



Source: Own elaboration.

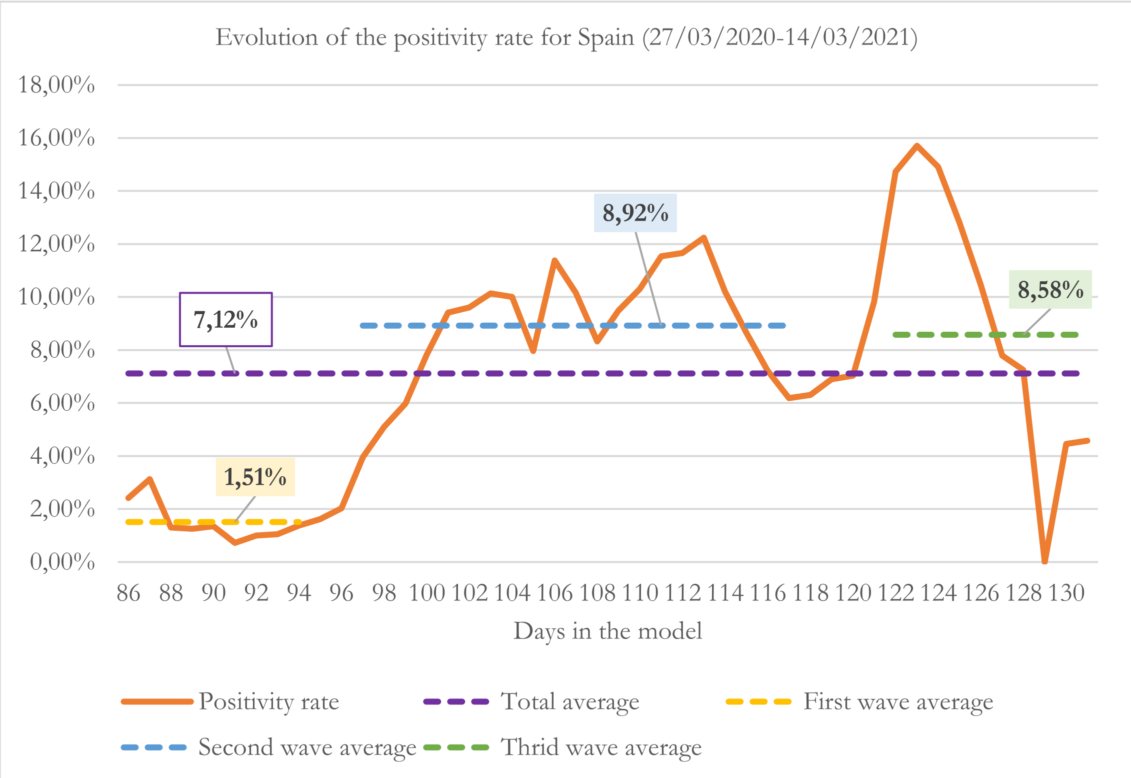
Spain. They deduced this value by observing the exponential growth of the virus at the beginning of the first wave (Casares & Khan, 2020).

Following the explanation of the contagion probability, once the lockdown kicks in and SoA is established, the contagion probability is reduced in 25% down to 0.016875 (1.6875%) mainly because the use of masks (along with other preventive measures) and it will never come back to the level observed in the pre-lockdown phase. With the end of the lockdown and the introduction of preventive measures by the government (such as the focus on teleworking, the control on social contacts, the still compulsory use of masks… and so on) and the arrival of the Summer (which makes people have outdoors interpersonal contacts) the contagion probability is steadily decreased up to day 236 in the model (21st of September 2020), two months after the end of the lockdown. This is the day before the beginning of Autumn. From that moment onwards, cold weather comes along with indoors meetings and the increase of the contagion probability. However, thanks to preventive measures mentioned before, it will only be increase up to 0.016875 (1.6875%). The same seasonal behaviour will be observed again in year 2021 from the beginning of Summer (day 509, 21st of Jun of 2021) up to the end of Autumn in day 692 (21st of December of 2021).

Besides, it is key to calibrate the testing rate for the case of Spain which is negatively correlated to the positivity rate. The positivity rate shows the proportion of infected individuals that have reported to have symptoms (or are reached because of recent direct contact with an infected individual) and are detected through a test. Therefore, the more infected individuals there are (the worse the pandemic gets) the higher the number of symptomatic people, the higher the probability of running a test on an infected induvial and the higher the positivity rate will be. However, if there are more infected individuals, the testing rate will be lower as the testing capacity to detect infected individuals (out of which most of them are asymptomatic) will decrease given the saturation of the model (too many infected people and the situation goes out of hands). Consequently, analyzing data on the positivity rate published by the ECDC (European Centre of Disease Prevention and Control) will provide the model with the highest attainable testing rate as I am about to show. Notice that the testing rate will be higher than the positivity rate when things are under control and lower with the worsening of the situation (Ministerio de Sanidad, Consumo y Bienestar Social, 2020).

First, it is important to determine the first day for the first COVID-19 test. In the next *figure 6* it can be observed the evolution of the positivity rate according to the ECDC since the 27th of March of 2020 (when Spain was already in the lockdown) up to the 14th of March of 2021 (even if I am only interested in data up to the 10th of March). I am going to assume that until the beginning of the lockdown no tests were carried out in Spain, even if the first cases where beginning to be detected, mainly because those tests represented such a low percentage of the population that can be disregarded. Notice that the first 12 days of the lockdown are not represented in *figure 6* because, as I have explained, they were not representative. Therefore, the initial value for the testing rate in the model will be zero at any time up to day 46 (lockdown day the 14th of March) of the model $v_0 = 0$ for $t < 47$.

Figure 6 The evolution of the positivity rate for Spain with data taken from ECDC.



Source: Own elaboration.

In *figure 5* above it can be observed the different lengths for the three waves of the pandemic and the average positivity rate in each of them. For the first wave (days 46-144 between 14/03/2020-21/06/2020) the average positivity rate is 1.51%, for the second wave (day 168-300 between 15/07/2020-24/11/2020) the average positivity rate is 8.92% and for the third wave (days 350-406 between 13/01/2021-10/03/2021) the average positivity rate is 8.58%.

During the first wave, the number of new cases was relatively low with respect to the rest of the waves and still many tests were carried out which explains a low positivity rate. However, as time goes by and with the arrival of the second and third waves, the number of known new cases increase much more than the number of tests brought about. These facts explain why the positivity rate increases with time. Consequently, from the beginning of the lockdown (14/03/2021) onwards, the testing rate will take a positive value that will daily depend on the number of known infected individuals according to equation (4). After having analysed the positivity rate, at most, I am going to stablish a testing of 12% (when the situation is at maximum control parameters) which is a bit higher than the average positivity rate during the second wave. This is quite reasonable to think given that the testing capacity considers the capacity to detect all the asymptomatic cases when with the positivity rate we are not truly taking all the asymptomatic cases into account but only those that have been in recent contact with an infected individual who happen to be tested. (ECDC, 2021)

Figure 7 Testing rate in the calibrated model.

Source: Own elaboration.

Figure 7 shows the evolution of the testing rate in the model simulation with the implementation of the testing function (4). Once the lockdown is stablished in day 46, tests are carried out in Spain for the first time and the daily testing rate begins to increase up to an upper bound of $v_0 = 0.12\ (12\%)$ (maximum control values). After the end of the lockdown and once the second wave kicks in (days 168-300), the testing rate in the model suffers from having more individuals infected. The testing rate decreases. The very same happens with the third way (days 350-406 when the number of known infected individuals increase even further) and Christmas time around days 300-400 in the model. Finally, with the decrease of the number of infected individuals and the beginning of the control recovery over the situation, the testing rate increases as it is observed when the vaccine arrives in Spain (day 365 with the first vaccinated individual). Still, behold that two more waves (forth and fifth ones) will be experienced in Spain at the beginning of the vaccination period, and these will come with the last time decreases in the testing rate.

Table 3 Daily and monthly vaccination rate.

Calculation of the daily and monthly rate of vaccinated individuals in Spain	
Fist day for providing the first vaccine dose	December 27th of 2020
Day for the first vaccinated induvial (once 2 doses have been provided to one individual)	January 28th of 2021
Last day for vaccination data (WHO)	March 10th of 2021
Number of passed days since the first vaccinated individual	**41 days** (from 28/01/2021 to 10/03/2021)
Number of doses administered	3,249,313 doses
Number of vaccinated individual (with 2 doses)	1,624,657 individuals
Spanish population	47,000,000 individuals
Daily proportion of the population vaccinated	**0.0843%**
Monthly proportion of the population vaccinated	**2.5293%**

Source: Own elaboration with data taken from the WHO.

Finally, with the introduction of the vaccine the daily rate of vaccinated individuals needs to be calibrated. According to the data published by the World Health Organization with the last update on the 10th of March of 2021, the daily rate of vaccination in Spain from the 28th of January (day 365) up to the 10th of March (day 406) has been $\varphi = \frac{2.5293}{30} = 0.000843\ (0.0843\%)$. This implies that over one month (30 days) a fraction of 2.5293%

of the population has been vaccinated which for a total of 41 days up to the 10^{th} of March accounts for 1,624,657 vaccinated individuals (for a total of 41 days since the first vaccinated individuals with two doses on day 365 in the model). All these dates and information is gathered in the previous *table 3*.

For the following period of time (day 406-730 between 10/03/2021 and 28/01/2021) for which there is yet not official data, the vaccination rate needs to be estimated. According to published information by the Spanish government authorities, the plan is to increase the daily vaccination rate and reach herd immunity by the end of summer. In addition, it seems reasonable for the vaccination rate to increase given the number of administered doses in other countries. According to WHO, the 23^{rd} of March Spain had already administered 16 doses per 100 of its population with one dose (16% of its population) while the US had administered 56 doses per 100 of its population (56% of its population as for the 01/04/21) and the UK had administered 52 doses per 100 of its population (52% of its population as for the 30/03/21).

In conclusion, disregarding updating dates differences, it can be said that in the US and the UK have had a three times faster vaccination speed compared to Spain. Given Spanish government aims to attain herd immunity by the end of summer (day 580 coinciding with the 31^{st} of August 2021), Spanish monthly vaccination rate must be of $\varphi = 7.5\ \%$ from the 10^{th} of March onwards. Notice that this implies tripling ongoing vaccination speed and having more or less the same monthly vaccination speed that in the USA and in the UK have been following up to the end of April (World Health Organization, 2021 and Raul Izquierdo, 2021).

Apart from this, it cannot be forgotten than only a proportion of the number of vaccinated individuals will turn out to become immune to the virus. This proportion is directly captured by the immunity rate in the model which indirectly depends on the efficacy rate of the vaccine. In the following *table 4* the different COVID-19 vaccines can be observed.

Up to the 10th of March there have been three vaccines in use in Spain: Pfizer-BioNTech or Vaxzevria (95% effective), Moderna (94.1% effective) and Johnson & Johnson/Janssen (67% effective). The main difference between the first two is that Moderna has an effectiveness of 86.4% on individuals older than 65 years old while Pfizer vaccine 61% effective on those aged above 70 years old. Besides, Johnson & Johnson vaccine requires only one dose, but it is only 66.2% effected on people older than 60 years-old which is quite a relative low value.

Taking all this into account, in *Table 4* displays an overall efficacy rate of 76.83% (64.08% on the elderly) for the COVID-19 vaccine setting equal weights on each one. However, if we consider that Moderna vaccines is mostly used on the elderly (also due to recent blood clots of the Astrazeneca vaccine on the elderly) and that Johnson & Johnson vaccine that requires an only dose will be implemented for the first time in April, the overall efficacy rate can be increased in the estimation. Consequently, the model will choose an efficacy rate of 80% (79.999% exactly) over the 324 days (day 730 minus day 406) for which the vaccination rate needs to be estimated. This implies that 20% of the total number of vaccinated individuals over these 324 days will turn out not to become immune to the virus and will still remain susceptible individuals. (EMA, 2020; A. C. EMA, 2021; D. EMA, 2021)

Table 4 Efficacy rates of the COVID-19 vaccines.

Efficacy rates of the COVID-19 vaccines		
Description	**Efficacy rates**	**Efficacy on the elderly**
Pfizer-BioNTech	95%	61%
Moderna	94.1%	86%
Johnson & Johnson/Janssen	67%	64.08%
Equal weights in the 3 vaccines	**76.83%**	**64.08%**

Source: Own elaboration with data taken from EMA.

This overall estimated efficacy rate is not directly determined in the model and must be captured by the immunity rate (θ). Therefore, an efficacy rate of 80% corresponds to an immunity rate of $\theta = 0.99698\ (99{,}698\%)$. This implies that from one day to the next nearly 0.3% of the vaccinated individuals lose their virus immunity.

To summarize the calibration section of the paper, all the parameters that have been calibrated for the evolution of the pandemic caused by COVID-19 in Spain can be found in the following table:

Table 5 Summary of the calibrated parameters in Spain to be implemented in the model.

Summary of the calibrated baseline model for Spain		
Description	**Parameter**	**Value**
Size of the Spanish population	N	47 million of people
Infection Fatality Rate (IFR)	λ_t	0.008 (0.80%)
Number of days for the incubation phase	T_i	5 days
Number of days for the outcome phase	T_p	25 days
Average number of days infected	$T^i + \frac{(T^p + 1)}{2}$	18 days
Asymptomatic proportion of individuals (in the outcome phase)	δ	0.4 (40%)
Number of interpersonal contacts in $t = 0$	y_0	20 people
Contagion probability in $t = 0$	α_0	0.0225 (2.25%)
Efficacy Rate (implicit in the model)	No parameter in the model	0.7999 (79.99%)
Daily Immunity Rate	θ	0.99698 (99.698%)
Maximum daily testing rate	v_0	0.12 (12%)
Daily vaccination rate	φ	0.000843 (0.0843%)

Source: Own elaboration.

ANALYSIS

Baseline scenario

The model is run and implemented in MATLAB. I have run it for two exact years (730 days) since the first infected individual in day one (29th of January 2020) up to day 730 (28th of January 2022). Over the two years of analysis, simulation results can be divided into two major periods: the first year of the pandemic (the lockdown together with the first, second and part of the third wave along with Christmas time) and the second year from the arrival of the vaccine onwards (the end of the third wave together with the fourth and the fifth waves).

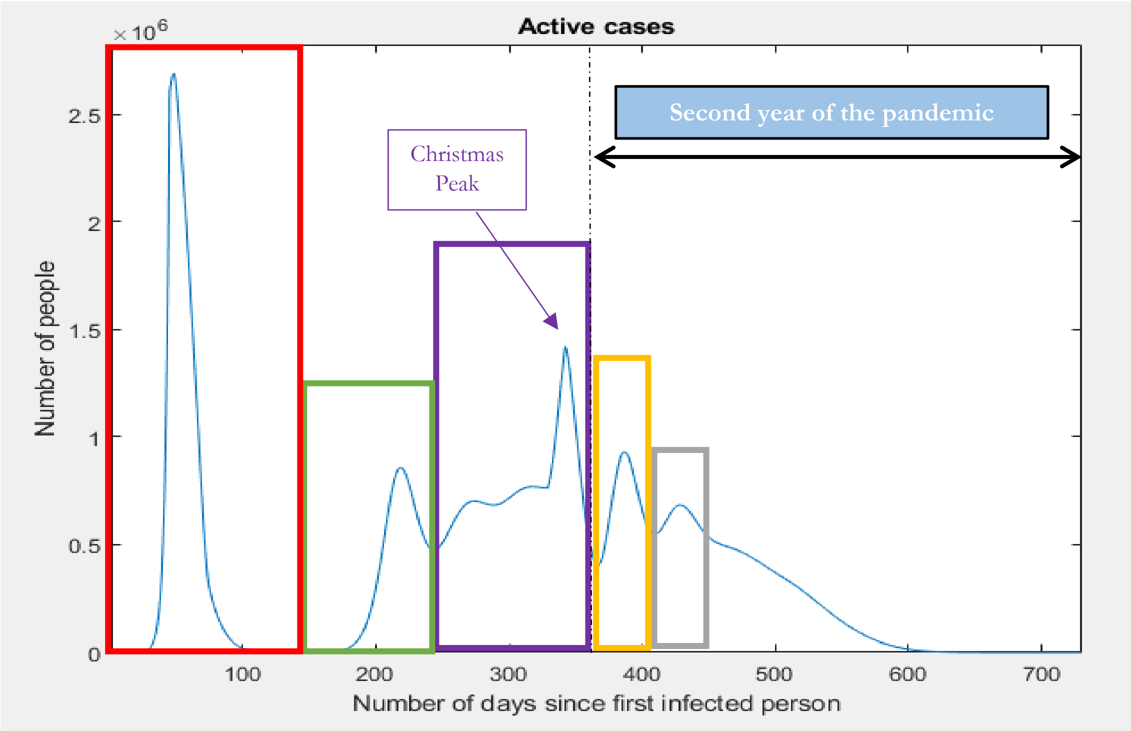
To sum up, the five waves of the pandemic I will be referring to are dated in the time periods gathered in *Table 6*. In *figure 8* the different waves are plotted and have been dated according to active cases data.

Table 6 Waves of the pandemic caused by COVID-19 in Spain.

Summary of dates for the five waves of the pandemic caused by COVID-19			
Name of the wave	**Colour in *figure 8***	**Days in the model**	**Real dates**
First wave	**RED**	**Days 46-144**	From 14th of March 2020 (14/03/2020) To 21st Jun 2020 (21/06/2020)
Second wave	**GREEN**	**Days 168-246**	From 15th of July 2020 (15/07/2020) To 1st of October 2020 (01/10/2020)
Third wave	**PURPLE**	**Days 247-365**	From 2nd of October 2020 (02/10/2020) To 28th of January 2021 (28/01/2021)
Christmas peak (third wave)	**Purple arrow**	**Days 327-355**	From 21st of December 2020 (21/12/2020) To 18th January 2021 (18/01/2021)
Fourth wave	**YELLOW**	**Days 366-406**	From 29th January 2021 (29/01/2021) To 10th of March 2021 (10/03/2021)
Fifth wave	**GREY**	**Days 407-450**	From 11th of March 2021 (11/03/2021) To 23rd of April 2021 (23/04/2021)

Source: Own elaboration.

Figure 8 Waves of the pandemic caused by COVID-19 in Spain under baseline model according to active cases.



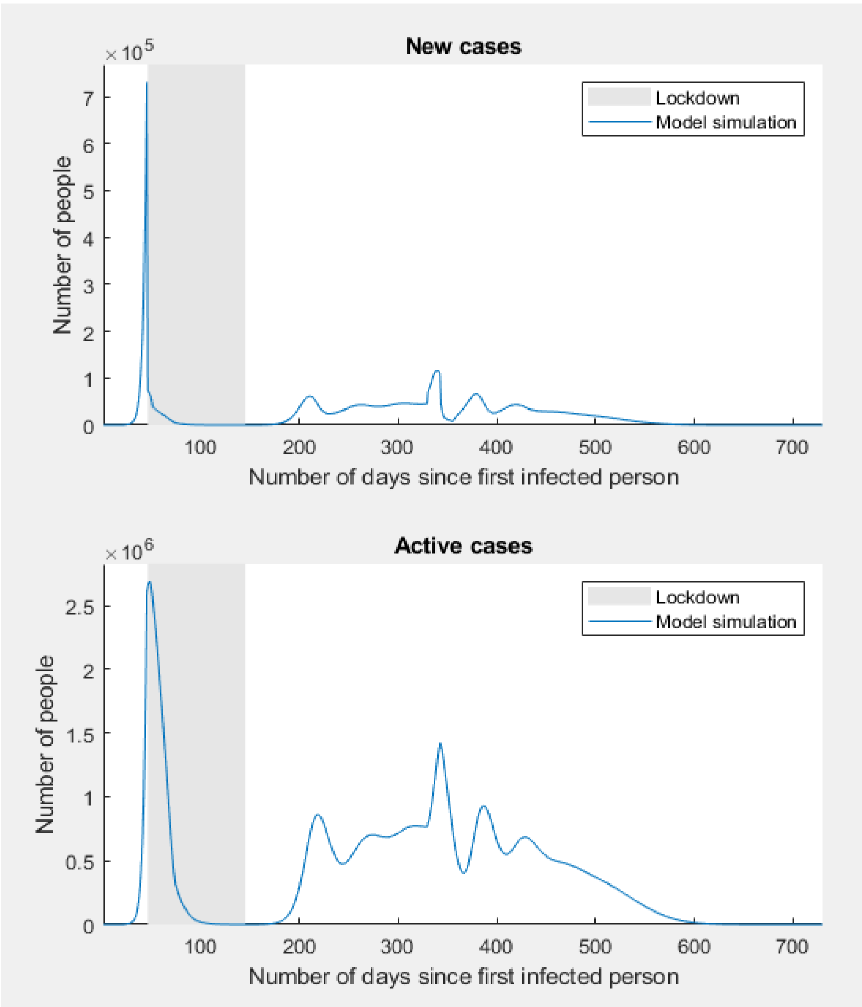
Source: Own elaboration.

Regarding the first year of the pandemic, up to the lockdown in day 46 the number of new daily cases skyrocketed from one in the first day of the model up to a maximum of 74.199 individuals the day when the SoA (State of Alarm) was implemented in Spain (day 46). Most of this exponential increase took place two weeks before the implementation of the lockdown (around day 30) when the situation went out of control. This was later called the first wave of the pandemic (see *figure 8* and *figure 9*).

During the beginning of the lockdown daily new cases dropped drastically, and cumulative active cases followed behind. Notice that the peaks corresponding to the different waves in cumulative active case in *figure 9* took place a few days after they had occurred in new cases figure. In day 144 when the lockdown is over (21st of Jun 2020), daily infected individuals are less than 2 and cumulative active cases are below 125 people which represents the lowest value for the period of analysis. As for the second and the third waves (days 168-246 and 247-365 respectively) along with the Christmas peak (around 327-355) the number of daily infected individuals is quite low regarding the first wave, but cumulative active cases happen to take an average level over this period (days 168-365) quite high and stable compared to the huge peak experienced during the first wave. All this is observable in the *figure 9*.

It must be highlighted that Spanish official data on the daily number of infected individuals up to the 10th of March of 2021 (last day for official data in the model) does not reflect the real number of infected individuals given the delay of Spanish institutions in reporting daily new cases, or due to individual decisions of not telling local authorities about having symptoms for the illness even if they are aware of it. Thus, I am not comparing daily or accumulated infected individuals in the model with the official Spanish data in *Figure 9*.

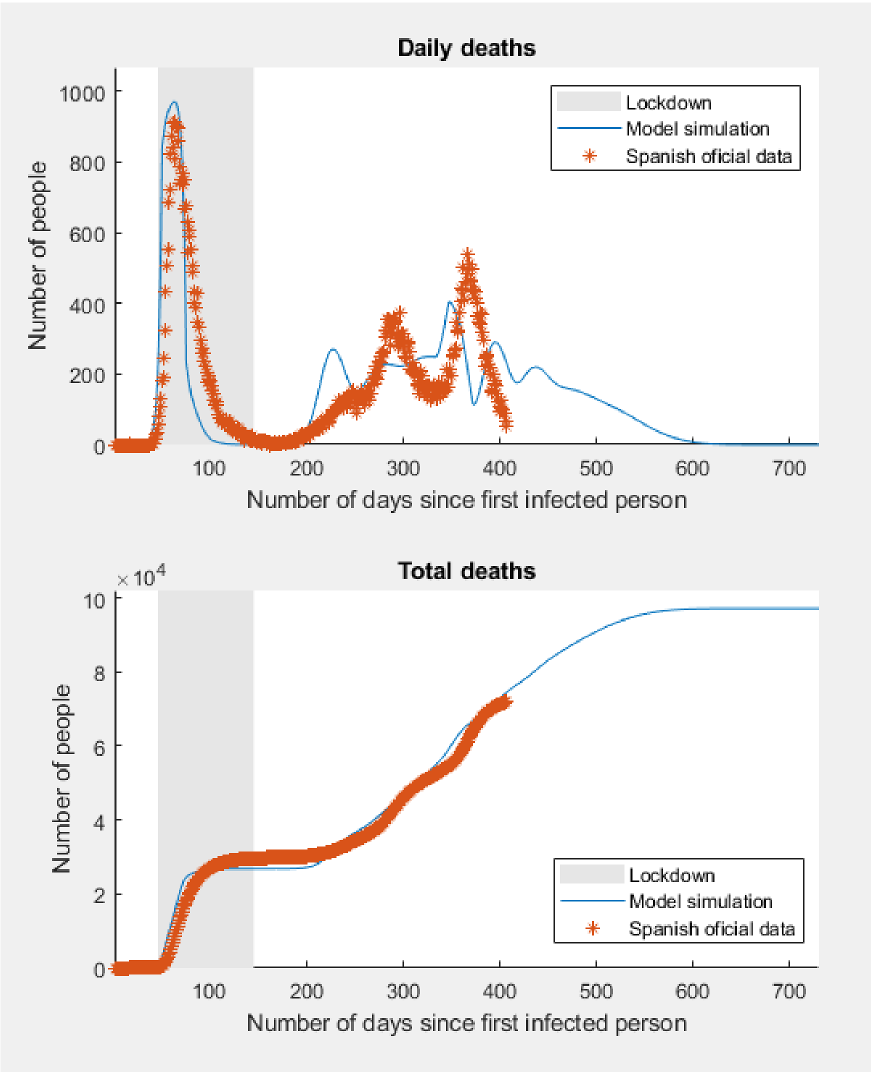
Figure 9 New and active COVID-19 cases under baseline model.



Source: Own elaboration.

Figure 10, daily and total deaths caused by the COVID-19 in Spain can be observed. It is remarkable how the model perfectly estimates the total number of deaths (between days 1-406) for the days for which I am taking Spanish official data (from the 29/01/2020 up to the 10/03/2021). However, regarding daily deaths, the model happens to anticipate what Spanish official data later shows which is especially noticeable with the arrival of the second and third waves as it can be observed in *figure 10*. This lag can be explained by the delays of reporting and recording deaths caused by COVID-19 in the Spanish territory. Still, during the first wave, the lag between the model simulation and the Spanish official data is much lower. Thus, during that time the model is a better fit for Spanish official data.

Figure 10 Daily and total deaths in the model simulation versus Spanish official data under baseline model.



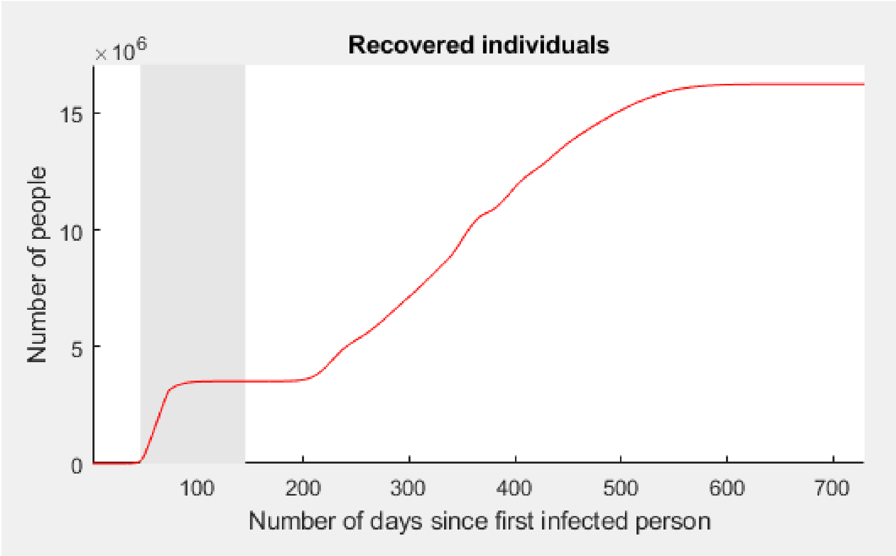
Source: Own elaboration.

It is remarkable that the greatest peak in daily deaths corresponds to the first wave of the pandemic and it is experienced the first week of the lockdown (shaded area). This one is consistent with the huge peak in new daily cases observed above in *figure 9* two weeks before the beginning of the lockdown. The highest daily deaths peak corresponds to day 63 (seventeen days after the beginning of the lockdown) with 970.54 individuals. Therefore, it is observable how a proportion of new cases will eventually die (according to the fatality rate) and will be observable in daily deaths graph around 2 weeks later (which is consistent with 18 average number of days an individual is infected).

In addition to this, regarding the steepness in total deaths curve it can be noticed that even if the greatest exponential increase in total deaths happened at the beginning of the first wave (steeper curve), a far greater number of individuals died during the second and third waves compared to the first one (comparison between top and bottom graphs in *figure 10*). This is reasonable given that second and third waves together have lasted longer than the first wave. Consequently, it can be said that the second and third waves were more fatal in terms of total deaths than the first wave even if the number of daily infected and daily death individuals were less as it can be observed in both *figure 9* and *figure 10*.

Having explained the evolution of daily and total new cases and deaths, it is interesting to observe what happens with the total number of recovered individuals. In *Figure 11* it can be noticed how the more infected individuals there are, not only the number of deaths increases but also the number of recovered individuals. Therefore, once the lockdown is stablished and daily cases drop to nearly cero it is not surprising that the total number of recovered individuals is maintained stable (from days 68 up to around day 200) until a bit after the beginning of the second wave. The same behaviour can be later observed with the arrival of the vaccine (similar to the lockdown effect). In this case, the number of recovered individuals is decreased more slowly along time given that the number of immune and vaccinated individuals goes adding up little by little which implies less infected people. As it can be observed, this takes time which goes from day 365 (first vaccinated individual) up to around day 580 when the total number of recovered individuals does not further increase and becomes stable.

Figure 11 Total recovered individuals under baseline model.



Source: Own elaboration.

The numerical summary of the results that have just been explained are observable in the following *table 7*:

Table 7 Main numerical effects of the pandemic in the calibrated model.

Summary of numerical results under the baseline model			
Variable of interest	**Day 144 (21/06/2021)**	**Day 580 (31/08/2021)**	**Day 730 (29/01/2022)**
Active cases	124.48	44,341	**1.31**
Recovered people	3,521,800	16,169,000	**16,236,000**
Total deaths	26,809	96,755	**97,126**

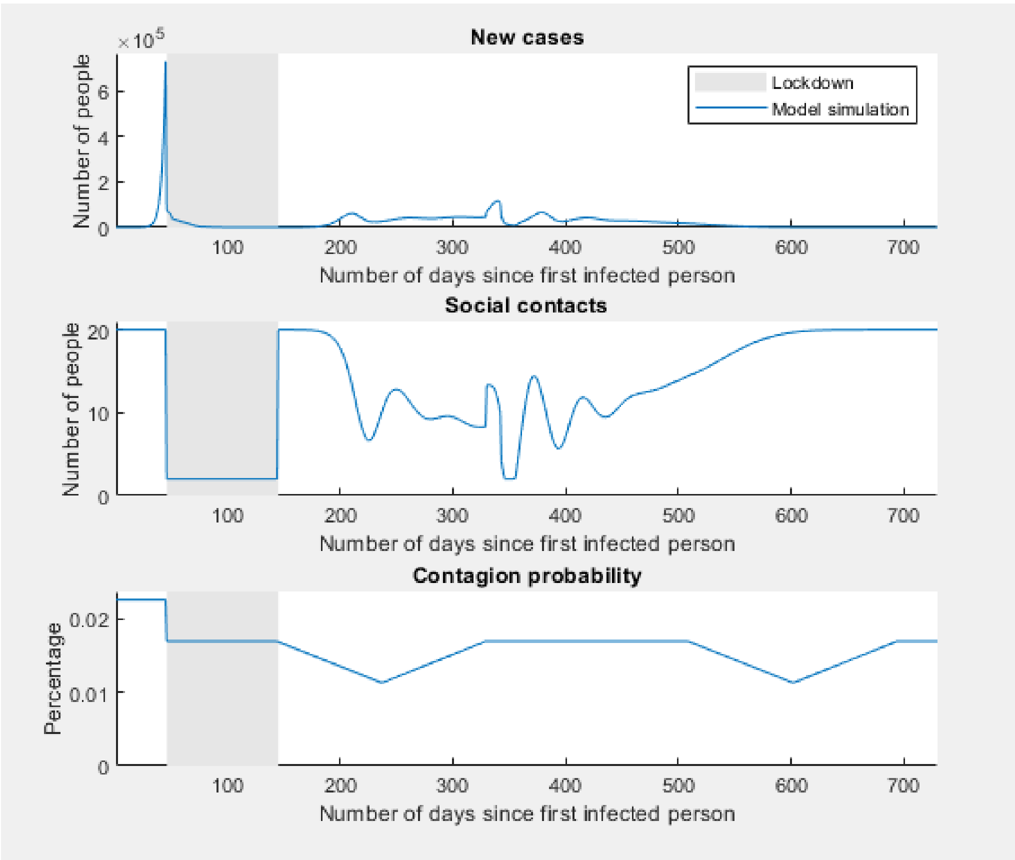
Source: Own elaboration.

The different effects on the number of infected and death individuals that have just been observed in *figure 9* and *figure 10* are explained by other parameters and equations in the model which evolution I am about to show and analyse.

First, new cases equation number two is determined by the contagion probability and the number of social contacts (equation 5) that can be observed in *figure 12* below. Every time the daily number of individuals increases too quickly government authorities will try to hold it back by decreasing the maximum number of allowed social contacts (mechanism under a pandemic with no vaccine). Besides, depending on the season of the year, the contagion probability will either ease the holding back of the spread of the virus (lower contagion probability in Summer) or it will speed up the transmission due to indoors interpersonal contacts in the Winter. Eventually the vaccine will arrive, and the virus spread will slow down (day 730, one year after the first vaccinated individual).

First, the establishment of SoA results in a sharp decrease of social contacts decreased down to minimum values (from 20 down to 2 individuals) and the contagion probability also decreased (from 2.25% down to 1.6875%) thanks to the establishment of compulsory mask and the social distancing during the lockdown. Once the lockdown is over and the pandemic is under control, social distancing measures go back to previous levels given that no restrictions are implemented. As for the contagion probably, the lockdown ends the 21st of June with the beginning of summer. Due to a greater amount of outdoors activity (more

Figure 12 Social distancing and the contagion probability effects on new cases equation under the baseline model.



Source: Own elaboration.

social distancing) and because of the heat (which happens to weaken the virus), the contagion probability begins to slowly decrease until the end of summer on the 21st of September(Mecenas et al., 2020).

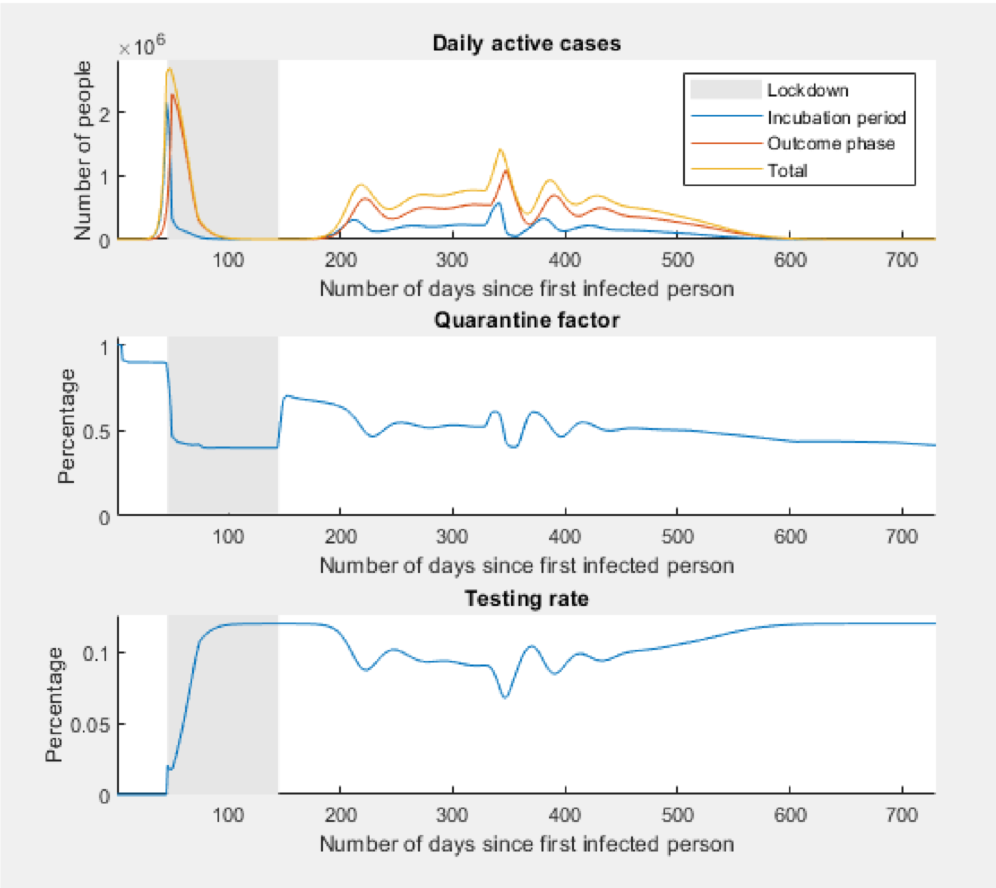
The end of Summer corresponds to the end of the second wave and the beginning of the third one which is partially explained by a higher contagion probability. During this time, government authorities try to hold this back with restrictions in the number of social contacts. With the beginning of Autumn (day 236, 21/08/2020) the contagion probability shows a constant slow increase due to having more indoors activity and it triggers a third wave that will take time to arrive because of government attempts to delay it reducing again social contacts (somewhere below 10 individuals). However, with the arrival of Christmas time, the government will allow family dinners with at most 6 individuals and two different family circles. This is translated in the model as an increase in 5 of the number of interpersonal contacts from the 24th of December of 2020 to the 6th of January of 2021 (days 330-342). This is directly translated into a peak in the number of new daily cases that is attempted to be decreased with more sever measures in social distancing. Eventually, the number of interpersonal contacts goes down to lockdown levels of 2 individuals.

Nevertheless, with the beginning of the new year, the first vaccinated individual will arrive in day 365 in the model (28th of January of 2021). Thanks to this, the fourth and last estimated wave, which will take place at the end of March, will be less detrimental. From that moment onwards, the effects of the vaccine will kick in because the number of daily new cases are going to be decreased without the need of establishing social distancing measures. Little by little, the number of social contacts will attain pre-pandemic values of 20 individuals. In fact, Spain will attain even faster pre pandemic social contact levels thanks to the decrease in the contagion probability (days 509-602) due to the arrival of the Summer in 2021.

Another key element determining new cases of COVID-19 is the quarantine factor. In the next *figure 13* it can be observed how for any given day, the higher the quarantine factor, the higher the daily number of infected individuals in either the incubation period or the outcome phase (higher overall infected individuals). Therefore, there is a lag of some days for the behaviour of the quarantine factor to be translated into daily active cases. This fact occurs because those infected individuals in the incubation period or in the outcome phase will know they are infected in the following days and only if they happen to be tested (because they either develop symptoms or because due to direct contact with a recent infected individual).

Besides, notice that those infected individuals in the incubation period always arrive before (blue line in top figure in *figure 13*) and that all of them will later belong to the infected individuals in the outcome phase (red line). In addition to this, as the total amount of days in the outcome phase can be up to 25 (5 times the number of days spent in the incubation period), total daily new cases equation (yellow line) behaves more similar to outcome phase red line rather than to incubation period blue line.

Figure 13 Quarantine factor and testing rate effects on new cases under the baseline model.



Source: Own elaboration.

On the other hand, before the establishment of the SoA, there was no testing rate in Spain that could detect the number of infected individuals in either of the phases which implied a quarantine factor of nearly $q_t \approx 1$. This happens because those infected individuals even with symptoms could not know they had the virus and were out there transmitting it to susceptible individuals (nearly the whole Spanish population). Nearly all uninfected individuals remained undetected, and the transmission accelerated dramatically.

With the beginning of the lockdown, the number of daily infected individuals decreases drastically as it can be observed in *figure 9*, and the testing rate reaches its maximum value $v_t = 12\%$ which has a positive impact lowering the quarantine factor to levels around $q_t = 0.4$ in *figure 13*. When talking about the testing rate, given that the testing rate is determined by past week data of the known infected individuals in equation 4, the testing rate is the first determinant having an effect first on the quarantine factor and a bit later on daily active cases. However, this lag happens for so few days that the effect cannot be genuinely appreciated in *figure 13*.

As for the second and third waves, the testing rate and the quarantine factor decrease whenever new cases increase and the other way around. During this time, the effect of the testing rate on the quarantine factor is not very notorious as in average the testing rate takes values around $v_t = 9\%$ for days between 168 and 365 (second and third waves). It can also be noticed that with the beginning of the vaccine the testing rate is more powerful given that the number of infected individuals increases less than in previous waves. Therefore, the vaccine attains to increase the testing capacity in Spain which slowly decreases the quarantine factor, and these helps lowering even further the number of daily new cases specially after the fourth wave around mid-March 2021 (after day 406 in the model).

To finish with the baseline analysis scenario, I am going to analyse the effects of the vaccine in *figure 14* and I am showing the summery of the results in *table 8*.

The vaccination period takes up one year from day 365 up to day 730 in the model. On the 28th of January 2021 (day 365), 76.66% of the population was susceptible of getting infected and almost all the remaining 23.34% of the population had recovered from the virus (total deaths up to then represented 0.14% of the population). By day 580 corresponding to the end of summer (31/08/2021), herd immunity of 70.09% is reached out of which 34.40% has gone through the virus (recovered individuals) and 35.69% has been vaccinated and has become immune to the virus. After two years since the first infected individual, herd immunity will be of 87.32% out of which 34.54% will have passed the virus and 52.77% will have been vaccinated and will have become immune to the virus. An important difference between day 580 and day 730 is that the percentage of those vaccinated individuals will be much higher in day 730 but the proportion of those individuals that will turn effectively immune will not differ that much from day 580.

Table 8 Effects of the vaccine under the baseline model in population shares.

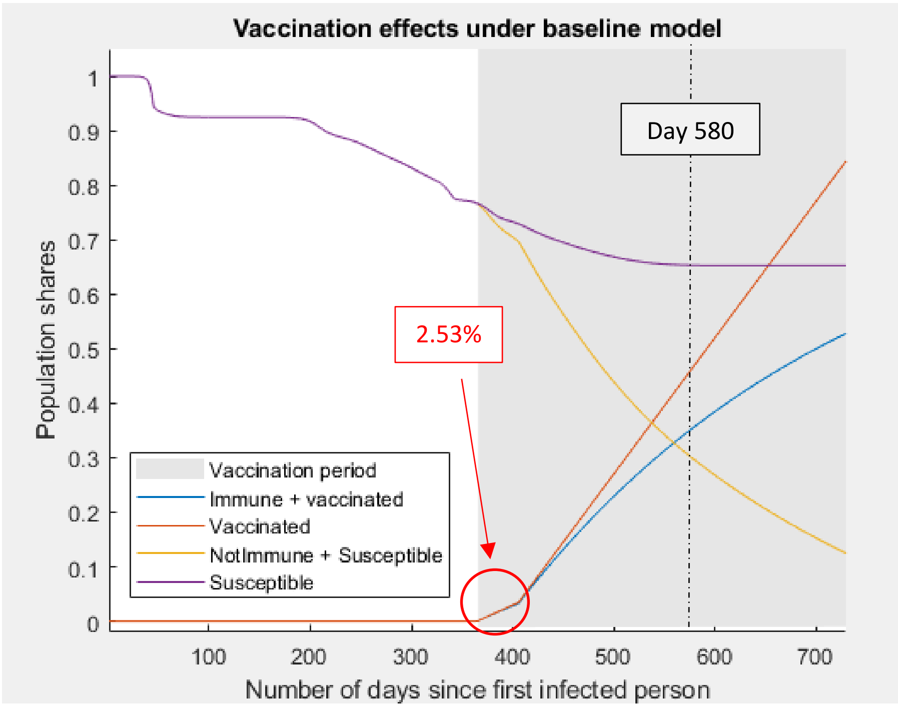
Summary of the effects of the vaccine under the baseline model				
	Data in % over total Spanish population	**Day 365 (28/01/2021) Beginning of the vaccination period**	**Day 580 (31/08/2021) End of summer (Herd immunity is reached)**	**Day 730 (28/01/2022) End of the estimation period (two years since the first infected individual)**
1	**Recovered share**	**22.333%**	**34.403%**	**34.545%**
2	**Death share**	0.1391%	0.20586%	0.20665%
3	**Vaccinated share**	0%	46.957%	84.457%
4	**Vaccinated+immune share**	**0%**	**35.69%**	**52.774%**
5	**Susceptible share**	76.662 %	29.607%	12.475%
6	**Herd Immunity (1) + (4)**	**22.33%**	**70.093%**	**87.319%**

Source: Own elaboration.

In *Figure 14*, the vaccination path is represented, and two vaccination peace can be observed. The first one corresponds to a monthly vaccination rate of 2.53% of the population (according to Spanish official vaccination data up to day 406) and the second one considers a 7.5% monthly vaccination rate (from day 406 to 730) which enables Spain to reach heard immunity by day 580 in the model (end of summer). As time goes by, those vaccinated and immune individuals represented by the blue line will drift apart from vaccinated ones (red line) due to the daily cumulative immunity rate that considers that every day there is a cumulative probability of not becoming immune to the virus after vaccination.

Eventually, for the whole period of analysis the effective rate of the vaccine will be 80% (79.99% precisely) which implies that 11.27% of the population will have been vaccinated but not immune by the beginning of 2022 (day 730).

Figure 14 Effects of the vaccine under baseline model.



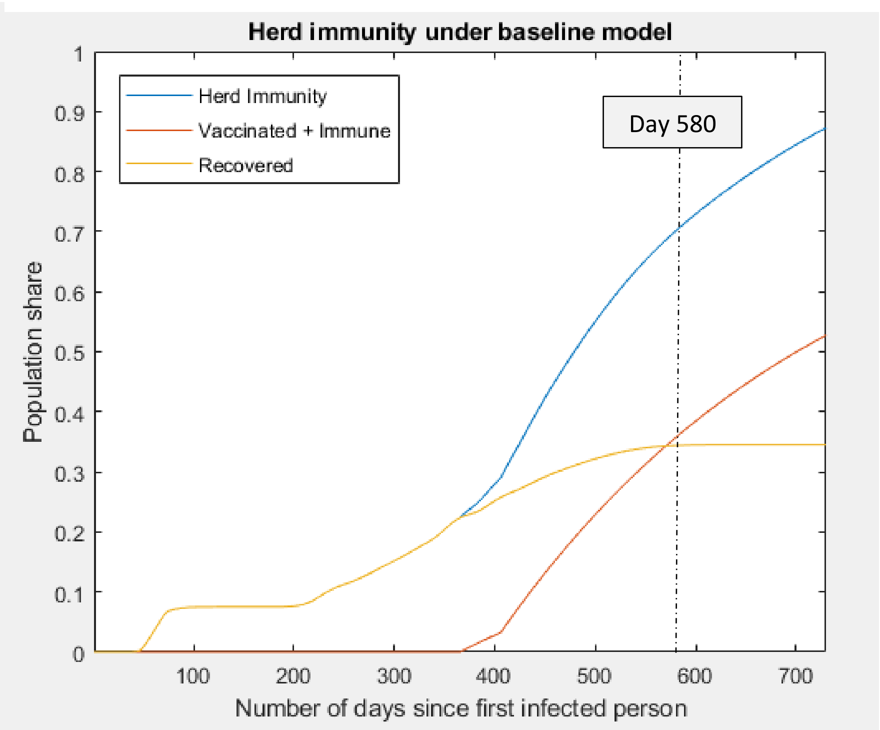
Source: Own elaboration.

Notice that the positive effects of the vaccine can be observed by the yellow line that shows the remaining susceptible individuals after taking out those vaccinated and immune ones. As it can be observed in *figure 14*, the difference between the yellow line and the purple line is precisely the blue line, or what is the same, the difference between susceptible individuals (purple line) and susceptible individuals after the vaccine (yellow line) is que number of those individuals that have been vaccinated and has become immune to the virus (blue line). As time goes by, more and more individuals will be vaccinated and immune. In fact, by day 580, 29.607% of the population will remain susceptible and by the beginning of 2022 (day 730), 12.475% of the population.

To finish with the analysis in the last *figure 15* of the baseline model I am going to show the evolution of the variable "herd immunity" which shows day by day which share of the population is immune to the virus. This variable considers the share of those vaccinated and immune to the virus as well as the share of those that have recovered from the illness ($herd\ immunity\ =\ vaccinated\ \&\ immune\ +\ recovered$). Before day 365 when the first person is vaccinated in Spain with two doses, only those that have got over the virus through previous infection (recovered individuals) can be immune to the virus. Therefore, for days 1-365 in the model, herd immunity coincides with the population share of recovered individuals (yellow line). However, once the vaccine is implemented, herd immunity differs from the population share of recovered individuals by the share of vaccinated and immune individuals (red line).

There are two interesting remarks to mention here. First, with the arrival of the vaccine the number of recovered individuals slows down (lower increase in the share of recovered individuals) and herd immunity increases nearly at the same rate as the number of vaccinated and immune individuals does (red and blue lines). Secondly, given that the passing of time makes the number of immune individuals among those vaccinated decrease (due to the cumulative effects of the immunity rate), the daily increasing returns of herd immunity in terms of population share decrease with time (lower steepness of blue line specially after

Figure 15 Herd immunity under baseline model.



Source: Own elaboration.

somewhere around day 500). This implies that the vaccine has a greater power in attaining herd immunity the faster the population is vaccinated.

Once that the functioning of the pandemic under the baseline model has been explained, it is time to go for the analysis of the model. The first part of the analysis focuses on reaching herd immunity and how the timing for this may be altered by changes in the immunity rate (related with the efficacy rate of the vaccine). Answers to important questions as the following ones will be given in this section: How important is for Spain to have a vaccine with high efficacy? Does a low efficacy rate imply too much delay on reaching herd immunity? How many lives can be saved due to a high efficacy rate?

The sensitivity to variations in the efficacy rate of vaccination

In the first part of the analysis of the model I am going to show the sensitivity of the just calibrated model for the case of Spain when the daily immunity rate represented by the parameter θ is increased and decreased from its baseline value of $\theta = 0.99698\ (99.698\%)$. This parameter has been determined in the model according to the efficacy rate that the vaccine is expected to have after 1 year from the day in which we consider the first vaccinated individual with two doses in Spain (day 365, 28/01/2021).

As I have already explained, the efficacy rate is not directly determined in the model but can be estimated dividing total number of vaccinated and immune individuals over 365 days (from day 365 up to the end of the analysis on day 730) by the total number of vaccinated individuals as follows: $mean\left(\frac{Nsv(366:730)}{Nv(366:730)}\right)$. By doing so I can obtain the immunity rate that needs to be implemented in the model that meets the expected efficacy rate of the vaccine over 1 year. In the calibration of the model, I decided to set a baseline efficacy rate of 80% (precisely 79.99%) for which the corresponding immunity rate is $\theta = 0.99698\ (99.698\%)$. Now, I am going to analyse the effects of an increase and decrease in 10% in the efficacy rate on the daily and total deaths as well as daily and total active cases. The summary of the changes to be implemented in MATLAB are presented in the following table:

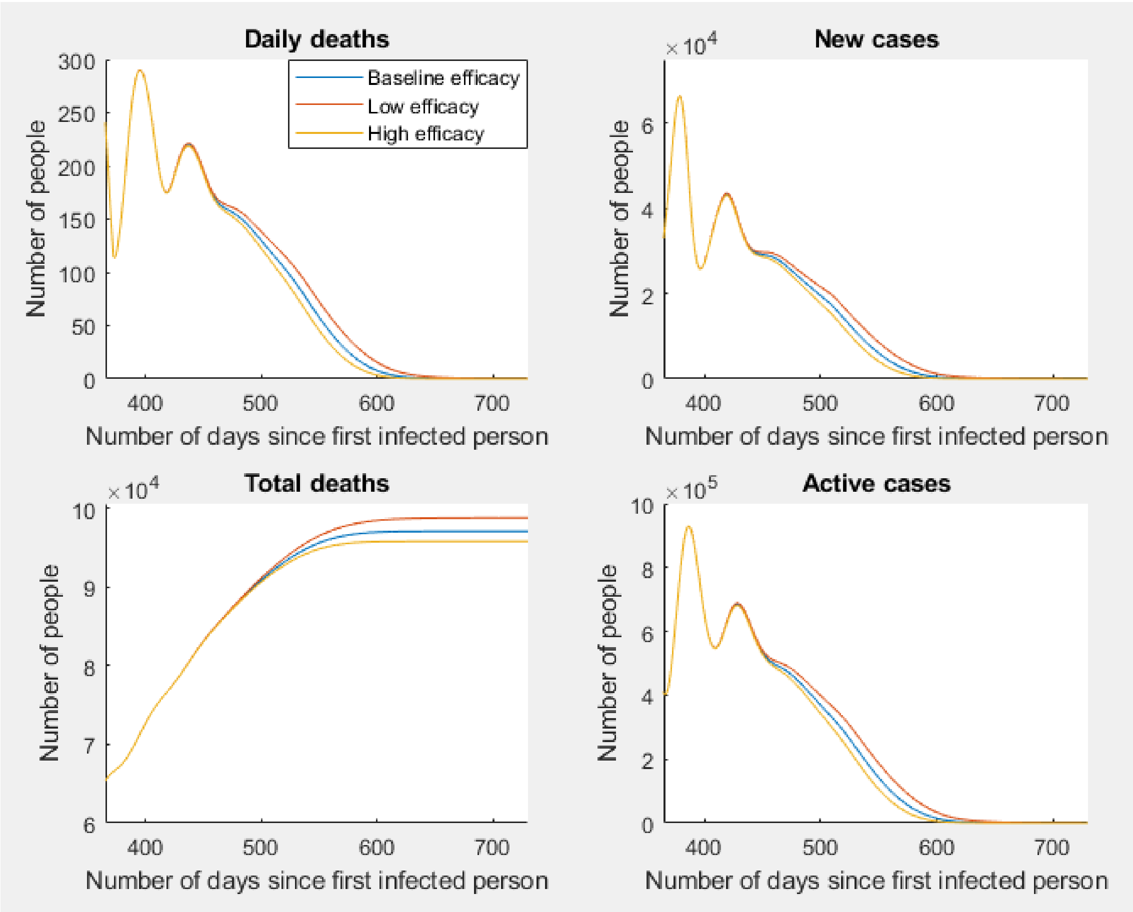
Table 9 Alternative designs of the immunity rate.

Changes to be implemented for the variations on the efficacy rate analysis		
Scenario	**Efficacy Rate (implicit over 1 year)**	**Immunity Rate (daily cumulative rate)**
Baseline scenario	80% (79.999%)	$\theta = 0.99698$ (99.698%).
High Efficacy Rate (increase in 10%)	90% (90.001%)	$\theta = 0.998649$ (99.8649%)
Low Efficacy Rate (decrease in 10%)	70% (70.00%)	$\theta = 0.994845$ (99.4845%)

Source: Own elaboration.

In the next *figure 16* the graphical effects of the previous changes can be observed. The graphs only show the effects for the different efficacy rates for the vaccination period of one year (days 365-730). For the case in which the efficacy rate is increased 10%, daily and total death curves as well as new and total active cases shift down (decrease) shown by the yellow curve. Just the opposite happens when the efficacy rate is decrease by 10%.

Figure 16 Simulated effects of alternative efficacy rates on vaccination variables.



Source: Own elaboration.

In terms of deaths and COVID-19 cases, there is not a big noticeable impact on the fourth (days 366-406) and fifth waves (days 407-450) of the model caused by changes in the efficacy rate. This happens because these two waves take place during the first months (March, April, and mid May 2021) since the implementation of the vaccine (a total of 84 days from days 365 to days 406) and the effects are more notorious as time goes by. Therefore, the values for deaths and active cases drift apart regarding the different efficacy rates somewhere after day 450 in the model, which is exactly the time after the fifth and last wave (23rd of April 2021). This has to do with the fact that the immunity rate is daily cumulative.

Consequently, the number of vaccinated and not immune individuals will be greater the more days have passed since the implementation of the vaccine and because of that, more individuals will be still susceptible implying that more individuals will get infected and die.

However, it will arrive a moment in time in which the number of vaccinated and immune individuals is quite high, the number of recovered individuals is also high and herd immunity will be at relatively high levels. When this happens, less people will get infected, and the number of deaths will also decrease. Consequently, the negative effects of having a low efficacy rate of the vaccine compared to a high efficacy rate will be diminished and the differences observed in *figure 16* between the yellow and the red line will decrease.

In the model, this time arrives somewhere around day 550 (1st of August 2021) and it is then when the three scenarios observed in *figure 16* begin to converge. Observe the next *table 10* in which herd immunity percentages are going to be gathered for five different time dates since the implementation of the vaccine. Notice that herd immunity differences for the two scenarios are quite low at the beginning but by the end of the period of analysis it implies a 25% difference. Besides, according to the WHO herd immunity is believed to arrive when 60-70% of the population gets immune to the virus and by day 550 in *table 9* both scenarios have already reached at least 60% of immunity.

Table 10 Herd immunity differences between low and high efficacy rates analysis in population shares.

Herd immunity difference between low and high efficacy rates analysis						
Efficacy rate	**End of the fourth wave Day 406 (10/03/21)**	**End of the fifth wave Day 450 (23/04/21)**	**Day 550 (01/08/2021)**	**Day 580 (31/08/2021)**	**Day 600 (20/09/2021)**	**Day 730 (28/01/2022)**
Low	**28.87%**	**41.48%**	**61.13%**	**64.79%**	**66.77%**	**75%**
High	**29.12%**	**43%**	**69.14%**	**75.37%**	**79.25%**	**101.76%**

Source: Own elaboration.

In the next *table 11* it can be observed how the difference between a low and a high efficacy rate in terms of deaths and COVID-19 cases is at greatest in day 550. At that exact time, there are 81,260 more active cases and 26,949 more daily deaths under a low efficacy rate scenario compared to a high efficacy rate one. Eventually, the convergence in terms of daily new cases and deaths ends around day 600 (20th of September 2021) with exceptionally low new cases and daily deaths values. This implies that total deaths stabilized and that the difference between day 600 and day 550 under the same scenario are very low.

Table 11 General effects of alternative immunity rates.

Timing for the difference regarding COVID-19 cases and deaths under efficacy rate analysis							
(number of people)	**Efficacy rate**	**End of the fourth wave Day 406 (10/03/21)**	**End of the fifth wave Day 450 (23/04/21)**	**Day 550 (01/08/2021)**	**Day 580 (31/08/2021)**	**Day 600 (20/09/2021)**	**Day 730 (28/01/2022)**
New cases	Low	33,341	29,754	8,535.6	3,027	1,159.4	11.20
	High	33,192	28,667	4,053.9	676	124.2	0
Difference		**149**	**1,087**	**4,481.7**	**2,395**	**1,035.2**	**11.20**
Active cases	Low	562,260	541,370	186,890	76,540	33,421	275.34
	High	561,130	529,110	105,630	24,003	5,795.7	0
Difference		**1,130**	**12,260**	**81,260**	**52,537**	**27,625.3**	**275.34**
Daily deaths	Low	236.88	194.93	71.667	32.10	15.146	0.12
	High	236.62	191.67	44.718	12.06	3.35	0
Difference		**0.26**	**3.26**	**26,949**	**20.04**	**15,142.65**	**0.12**
Total deaths	Low	74,304	83,210	96,568	98,068	98,518	98,849
	High	74,300	83,131	94,886	95,658	95,794	95,832
Difference		**4**	**79**	**1,682**	**2,410**	**2,724**	**3,017**

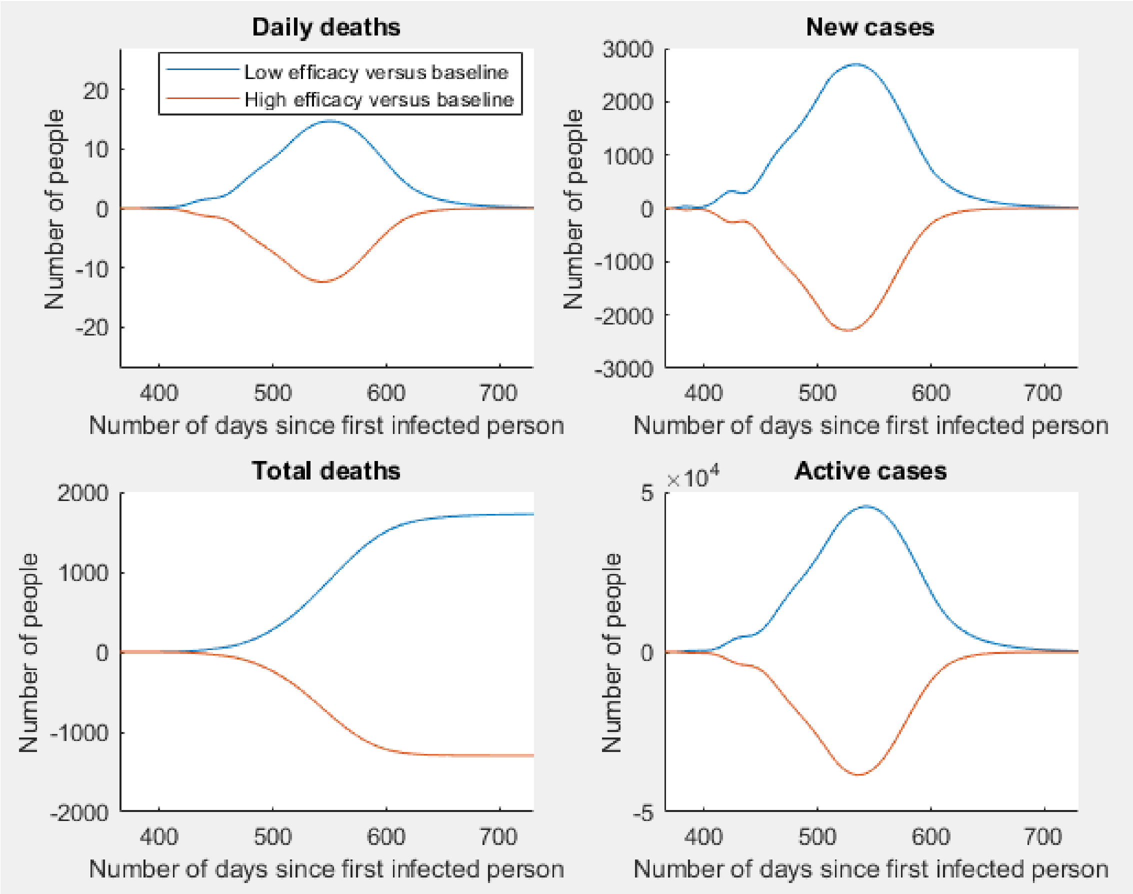
Source: Own elaboration.

Nevertheless, because in *figure 16* the differences with low and high efficacy rates scenarios towards the baseline model are not that observable, in the next *figure 17* I have calculated the difference in the effects of high and low efficacy rates versus the baseline scenario for the very same four variables of analysis: daily deaths, total deaths, new cases and total active cases.

It is observable how daily deaths curves are delayed (in around 18 days given the duration of the illness) in comparison with new cases curves as the last ones take place before.

Besides, the positive effects in the decrease of daily deaths and new cases take place earlier when having a high efficacy rate compared to a low efficacy rate.

Figure 17 COVID-19 incidence and deaths under different efficacy rates.

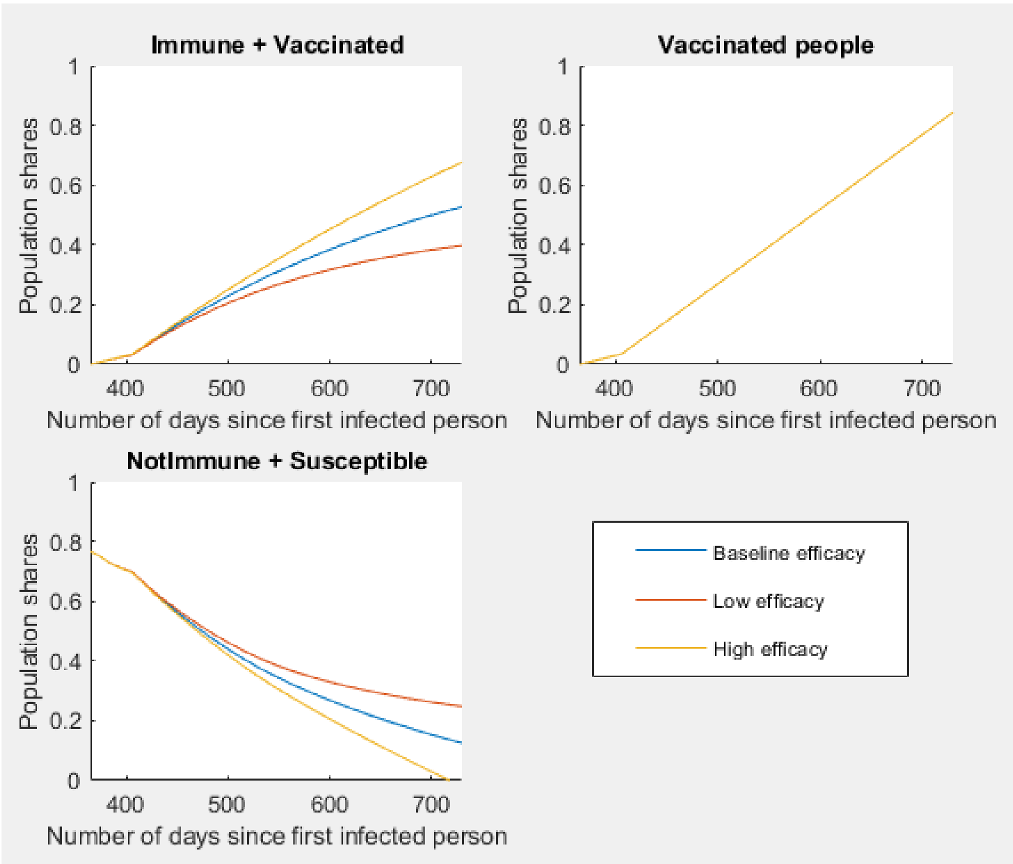


Source: Own elaboration.

Overall, the effects are similar but with opposite directions for both cases in which the efficacy rate is increased and decreased in 10%. However, it can be noticed how the effects of a low efficacy rate of 10% are a bit more severe in terms of total active cases and total deaths. According to daily deaths, the change is not very notorious as at most daily deaths will increase at most in 15 people or decrease in 13 people. This has an effect on total deaths of an increase of around 1,700 people when the efficacy rate is low and a decrease of 1,300 people when the efficacy rate is high. When looking at new cases figure, daily new cases are increased at most in 2,700 people (with low efficacy rate) and decreased in 2,300 people (with high efficacy rate). This affects total active cases that will increase at most in around 45,000 individuals or decrease in 38,000 individuals.

Finally, it is time to have a lock at vaccination variables and their evolution under the different scenarios (*figure 18*). In general terms, regarding vaccinated people, a change in the immunity rate has no effect on the number of vaccinated individuals as every day a fixed proportion of the population is vaccinated (same yellow line for the three scenarios). However, if we have a look at the share of the population that is vaccinated and becomes immune to the virus at the end of the analysis (day 730) it can be noticed that the difference towards the baseline model is +15% (for high efficacy rate) and -13% for low efficacy rate. Finally, those not immune and susceptible represent almost 25% of the population for a low efficacy rate while there are none of them under high efficacy rate scenario.

Figure 18 The effects of low and high efficacy rates on vaccination variables in population shares.



Source: Own elaboration.

Apart from this, it is quite interesting to observe what has happened to the herd immunity variable over this year of analysis under the different scenarios. I am going to take the data already presented in *table 10* and I am going to further classify it into two more categories: the proportion of those recovered individuals over the population, and the proportion of those vaccinated and immune to the virus over total population. I am also including day 730 for the 28th of January 2022 to observe which is the value of herd immunity at the end of the year since the first vaccinated individual on day 365. This information is gathered in the following *table 12*:

Table 12 Herd immunity under alternative vaccination efficacy rates in population shares.

Herd immunity for the three scenarios under alternative vaccination efficacy rates.							
% of the Spanish population	**Efficacy rate**	**End of the fourth wave Day 406 (10/03/21)**	**End of the fifth wave Day 450 (23/04/21)**	**Day 550 (01/08/2021)**	**Day 580 (31/08/2021)**	**Day 600 (20/09/2021)**	**Day 730 (28/01/2022)**
Herd Immunity	Low	28.87%	41.48%	61.13%	64.79%	66.77%	**75%**
	Baseline	29.01%	42.30%	65.22%	70.09%	72.94%	**87.32%**
	High	29.12%	43.00%	69.14%	75.37%	79.25%	**101.76%**
Recovered individuals	Low	25.76%	29.19%	34.33%	34.91%	35.08%	**35.21%**
	Baseline	25.76%	29.17%	33.98%	34.40%	34.50%	**34.54%**
	High	25.76%	29.16%	33.68%	33.98%	34.03%	**34.05%**
Vaccinated + immune	Low	3.11%	12.29%	26.80%	29.88%	31.68%	**39.79%**
	Baseline	3.25%	13.13%	31.24%	35.69%	38.44%	**52.77%**
	High	3.36%	13.84%	35.46%	41.39%	45.22%	**67.71%**

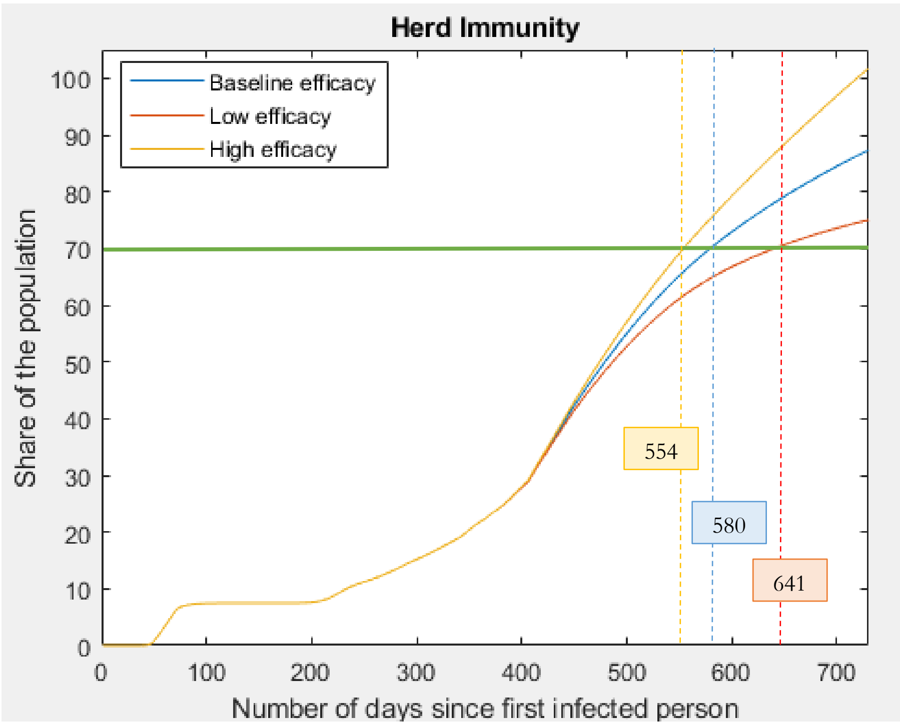
Source: Own elaboration.

As it can be observed, the differences in terms of herd immunity for the three scenarios are not notorious up to day 550 when a herd immunity of 60% is already reached in all the three cases. This timing corresponds to the moment in which the differences observed in terms of COVID-19 cases and deaths in *table 12* and *figure 17* are at their greatest values. Besides, the observed differences in herd immunity come from the differences in the proportion of vaccinated and immune individuals. High efficacy rate implies a high daily immunity rate which is translated as more vaccinated and immune individuals and a higher herd immunity specially from day 550 onwards.

At the end of the period 365-730, low efficacy rate implies having around 12.33% less of the population immune to the virus (74.993%-87.319%) compared to the baseline model and high efficacy rate implies having 14.44% more of the population vaccinated. In fact, with high efficacy rate Spain would reach 100% herd immunity in day 719 (17th of January 22), 11 days before the last day in our model. In conclusion, with high efficacy rate the whole Spanish population would be immune to the virus in less than a year, and 66% of the population would have been vaccinated and immune to the virus.

To finish with the analysis of the effects of different efficacy rates on herd immunity, in the following *figure 19* I am showing a graphical representation for the different values herd immunity takes every day under the different efficacy rates scenarios.

Figure 19 Reaching herd immunity under different efficacy rates scenarios.



Source: Own elaboration.

Given that herd immunity is set to be achieved when 70% of the population is immune to the virus, in the following *table 13* I have computed the differences of low and high efficacy rates towards the baseline model. As it can be observed, it is more harmful for the Spanish society to have a low efficacy rate as it delays reaching herd immunity by 61 days (around two months) than to have a high efficacy rate that speeds up herd immunity timing by only 26 days (less than a month).

Table 13 Calendar effects for reaching herd immunity of 70% under alternative efficacy rates.

Timing differences in reaching herd immunity of 70% under different efficacy rates				
Description	**Efficacy rate**	**Immunity rate**	**Herd Immunity timing**	**Speed difference towards baseline speed**
Low efficacy rate (-10%)	70%	$\theta = 0.994845$ (99.4845%).	31st of October 2021 **(Day 641)**	**+ 61 days delay** **+10.52% delay** **(around 2 months delay)**
Baseline efficacy	80%	$\theta = 0.99698$ (99.698%).	27th of December 2021 **(Day 580)**	**0**
High efficacy rate (+10%)	90%	$\theta = 0.998649$ (99.8649%).	5th of August 2021 **(Day 554)**	**-26 days in advance** **-4.48% speed up** **(around less than a month earlier)**

Source: Own elaboration.

In conclusion, it is clear by now that it takes longer for the model to attain herd immunity under low efficacy rate (day 641) rather than under high efficacy rate (day 554). Besides, it is more harmful for the Spanish society to have a low efficacy rate as it delays reaching herd immunity by 61 days (around two months) than to have a high efficacy rate that speeds up herd immunity timing by only 26 days (less than a month). Recall that for both low and high efficacy rates the difference in terms of efficacy rate towards the baseline model is +/- 10% of efficacy in the vaccine.

Once we have studied the effects of the immunity rate on reaching herd immunity it is time to do the same with the vaccination speed. In the next and last part of the analysis section of the paper the importance of a fast monthly vaccination speed is going to be assessed. Does a low efficacy rate delay more reaching herd immunity than a low vaccination speed? Which one should be considered a priority and why? These are some of the questions that will be answered in the following part.

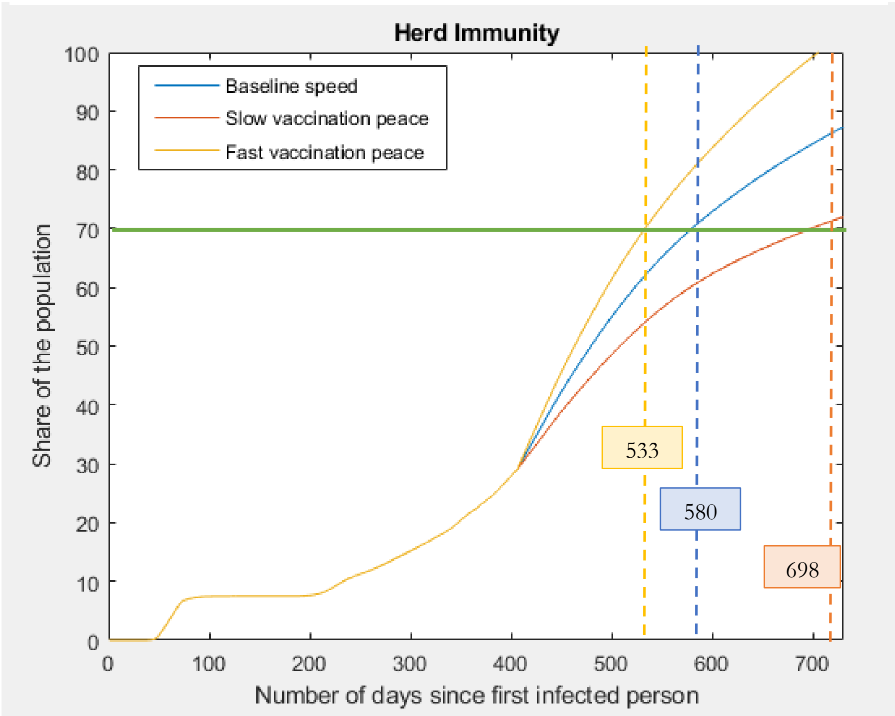
The effects of the vaccination speed

In this second part of the analysis, I am going to analyse the consequences of having a faster or a lower daily vaccination peace for the case of Spain. In the calibration part of the paper, I explained that from the first day of vaccination in Spain (28thof January 2020) up to the last day for which I take Spanish official data (the 10th of March), the monthly vaccination rate in Spain has been 2.5293% of the population. Besides, I estimated that from that moment onwards (up to the end of the analysis in day 730) the monthly vaccination peace for the baseline model was going to be 7.5% in order to attain herd immunity by the end of summer 2021 (day 580). Now, in this second section of the analysis I am going to present the effects of having a monthly vaccination different from 7.5% from the 10th of March onwards (days 406-730 in the model). I am going to analyse the evolution of the pandemic if the monthly vaccination rate increases to 10% of the population (+2.5%) or it decreases to 5% of the population (-2.5%).

First of all, it is important to have in mind that a slower monthly vaccination rate implies reaching herd immunity later. This comes along with the increase in daily new cases and deaths which, by the definition of the equations in the model, it will also decrease the number of recovered individuals as more people will get infected. Besides, it is key to realise that the capacity of the current vaccine to give immunity to individuals weakens with time due to the increasing probability for the immunity rate of the vaccine to decrease with time. Therefore, the effects in terms of daily deaths and new cases that I am about to analyse in the next *figure 20* will be greater and more detrimental for the Spanish economy for the case of having a low vaccination rate.

Before going any further, in *figure 20* and *table 14* I am going to present the summary of the effects on the timing for reaching herd immunity for the three different vaccination speed scenarios. Recall that herd immunity is reached whenever 70% of the population is immune to the virus because it has either got over it through past infection or because it has been vaccinated and has turned immune to the COVID-19. Day 406 in the model corresponds to the 10th of March which is the last day for which I have considered Spanish official data on vaccination speed, and thus, it is the beginning of the vaccination speed scenarios analysis that I am bringing about now. Notice that this day coincides with the beginning of the fifth wave in the model (days 407-450).

Figure 20 Reaching herd immunity under different vaccination speed scenarios.



Source: Own elaboration.

Table 14 Herd immunity in alternative scenarios of vaccination speed analysis in population shares.

Herd immunity in the alternative scenarios of vaccination speed analysis						
% of the Spanish population	**Efficacy rate**	**End of the fourth wave Day 406 (10/03/21)**	**End of the fifth wave Day 450 (23/04/21)**	**Day 550 (01/08/2021)**	**Day 600 (20/09/2021)**	**Day 730 (28/01/2022)**
Herd Immunity	Low	29.01%	38.93%	56.55%	62.34%	**72.00%**
	Baseline	29.01%	42.30%	65.22%	72.94%	**87.32%**
	High	29.01%	45.67%	73.93%	83.81%	**103.12%**
Recovered individuals	Low	25.76%	29.23%	35.02%	36.11%	**36.41%**
	Baseline	25.76%	29.17%	33.98%	34.50%	**34.54%**
	High	25.76%	29.12%	32.98%	33.16%	**33.16%**
Vaccinated + immune	Low	3.25%	9.70%	21.53%	26.23%	**35.59%**
	Baseline	3.25%	13.12%	31.24%	38.44%	**52.77%**
	High	3.25%	16.55%	40.95%	50.65%	**69.96%**

Source: Own elaboration.

As it can be observed, under the three vaccination speed scenarios herd immunity is reached by the end of the analysis on day 730 (28th of January 2022). In fact, the total share of vaccinated individuals for a fast vaccination speed is already the whole Spanish population by day 696 (25/12/2021) but the proportion of the population that is immune is around 65% at that time. Still, by the end of the two years of analysis, under a fast vaccination rate herd immunity would be more than 100% while under a low vaccination rate, we would have just passed the so claimed 70% of herd immunity (72.00% precisely).

Notice that the differences in terms of herd immunity are originated by the differences in the share of those vaccinated an immune individual, which difference aggravates with time. It is observable how during the first days of the different vaccination speed scenarios the yellow line (fast speed) does not differ to much from the baseline model compared to the red line (low speed). In fact, this has to do with the idea that as few days have passed since the first vaccinated individual, the number of individuals that have been vaccinated but are not immune are still low (due to the efficacy rate of the vaccine).

Table 15 Calendar effects for reaching herd immunity of 70% for alternative vaccination speed scenarios.

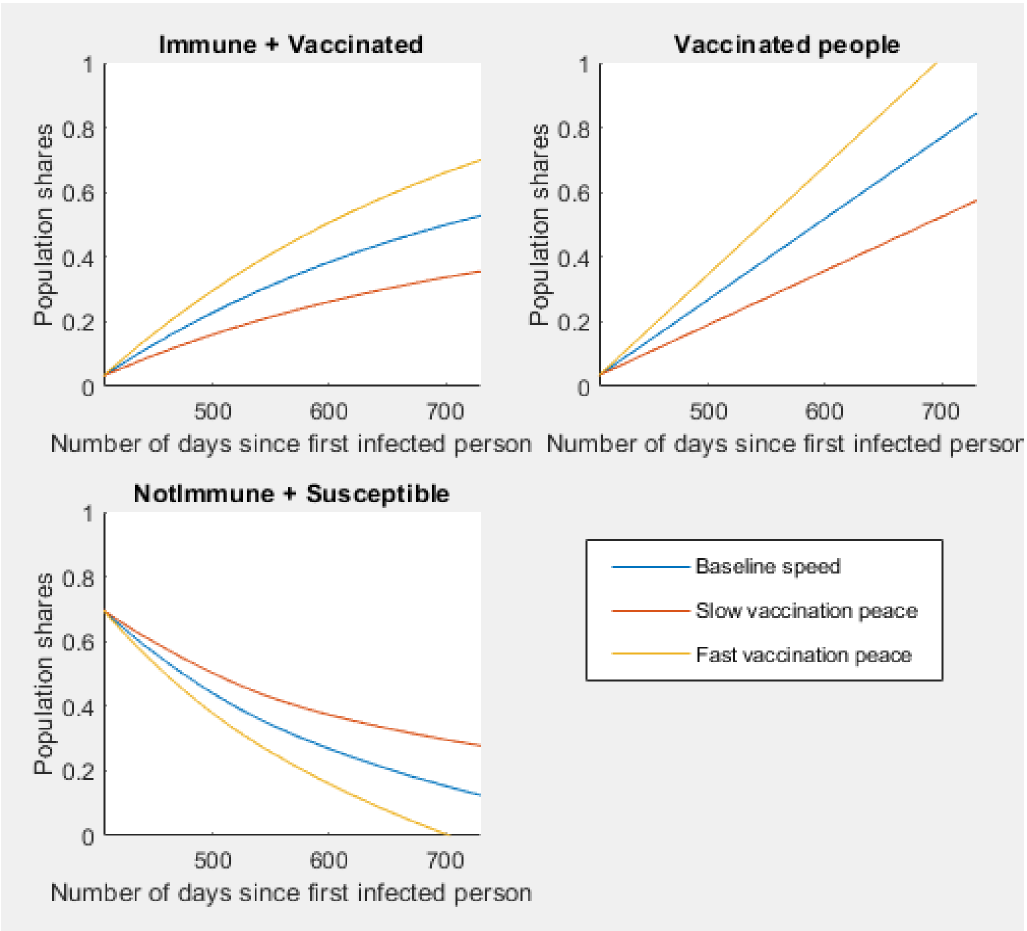
Timing differences in reaching herd immunity of 70% for alternative vaccination speed scenarios			
Description	**Monthly vaccination speed**	**Herd Immunity timing**	**Speed difference towards baseline speed**
Slow vaccination peace	5%	31st of August 2021 (Day 698)	**+ 118 days delay** **+20.34% delay** **(around 4 months delay)**
Baseline speed	7.5%	27th of December 2021 (Day 580)	**0**
Fast vaccination peace	10%	15th of July 2021 (Day 533)	**-47 days in advance** **-8.10% speed up** **(around 1.5 months earlier)**

Source: Own elaboration.

In *table 15* the same idea can be deduced. It takes longer for the model to reach herd immunity under low vaccination speed (day 698) compared to fast vaccination speed (day 553). In conclusion, under a slow vaccination peace the model experiences a delay reaching herd immunity of 118 days, which compared to the enhancement in reaching herd immunity when having a fast vaccination peace is only 47 days of speed up. This is directly explained by the immunity rate that increases to a greater peace the number of vaccinated but not immune individuals as more days go by since the beginning day for the analysis (day 406, 10th of March). Therefore, as the speed difference between both scenarios is 2.5% towards the baseline model, having a lower vaccination speed of 5% compared to a faster vaccination speed of 10% has a greater impact on herd immunity timing.

In the following *figure 21* the different vaccination variables and their time evolution can be observed. Overall, a faster vaccination peace implies having a greater share of the population vaccinated which is most importantly translated into a higher proportion of vaccinated and immune individuals and a lower one for those not immune and still susceptible individuals.

Figure 21 Simulated effects of alternative vaccination speeds on vaccination variables in population shares.



Source: Own elaboration.

It is quite interesting to observe that even if a higher or lower monthly vaccination rate might not have a direct impact on the efficacy rate of the vaccine, this last one is slightly altered, and it is the immunity rate in the model. In the following *table 16* the attained efficacy rate for the different vaccination speeds is observable. The reasoning behind is that given that the population share of those immune and vaccinated individual changes as well as the proportion of vaccinated individuals, the efficacy rate is also altered.

Table 16 Monthly vaccination rate impact on the efficacy rate of the vaccine.

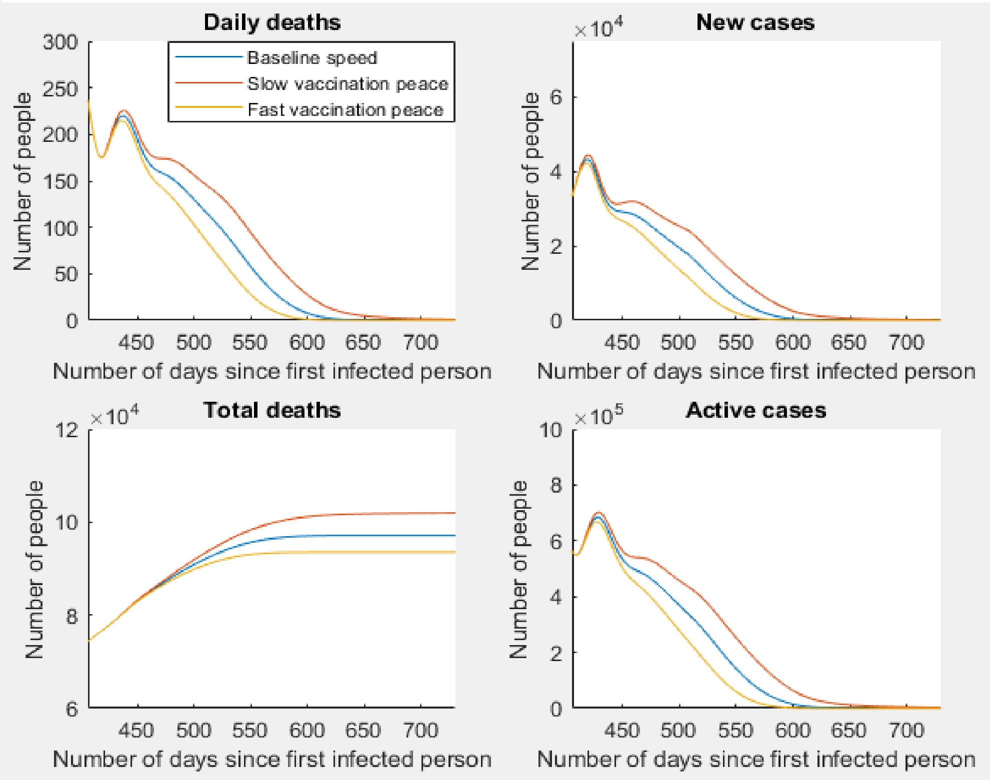
The impact of changes in the monthly vaccination rate on the efficacy rate of the vaccine	
Description	**Efficacy Rate**
Slow vaccination peace	**79.35%**
Baseline peace	**80% (79.999%)**
Fast vaccination peace	**80.36%**

Source: Own elaboration.

In the next *figure 22* the difference on daily and total deaths and active cases for the three scenarios of analysis can be observed: a baseline speed of 7.5% monthly population rate (blue line), a slow vaccination peace of 5% monthly population rate (red line) and a fast vaccination peace of 10% monthly population rate. In *figure 21* I have taken data from day 406 corresponding to the last day for available Spanish data (10^{th} of March) up to day 730 (28thof January 2022). Overall, new cases and total deaths take greater values with a low vaccination rate and deaths experience a delay of some days in comparison with new cases.

As far as the difference observable in *figure 22* in terms of deaths are concerned, *table 16* shows that total deaths have been 101,980 people when the vaccination speed was low and 93,521 when the vaccination speed was high. Therefore, the model estimates that having a fast vaccination rate compared to a slow one can save 8,459 people from dying from the 10^{th} of March 2020 up to the 29^{th} of January 2022 (a period of 324 days). Still, the difference in total number of deaths towards the baseline scenario for a low vaccination rate is much higher (4,854 deaths more) as compared to the case with fast vaccination rate (3,605 deaths less). This difference can be better observed in *figure 23* in which I have computed the difference of each scenario towards the baseline model.

Figure 22 Effects of changing vaccination peace on daily and total deaths and active cases.



Source: Own elaboration.

Table 17 General effects of different vaccination speeds.

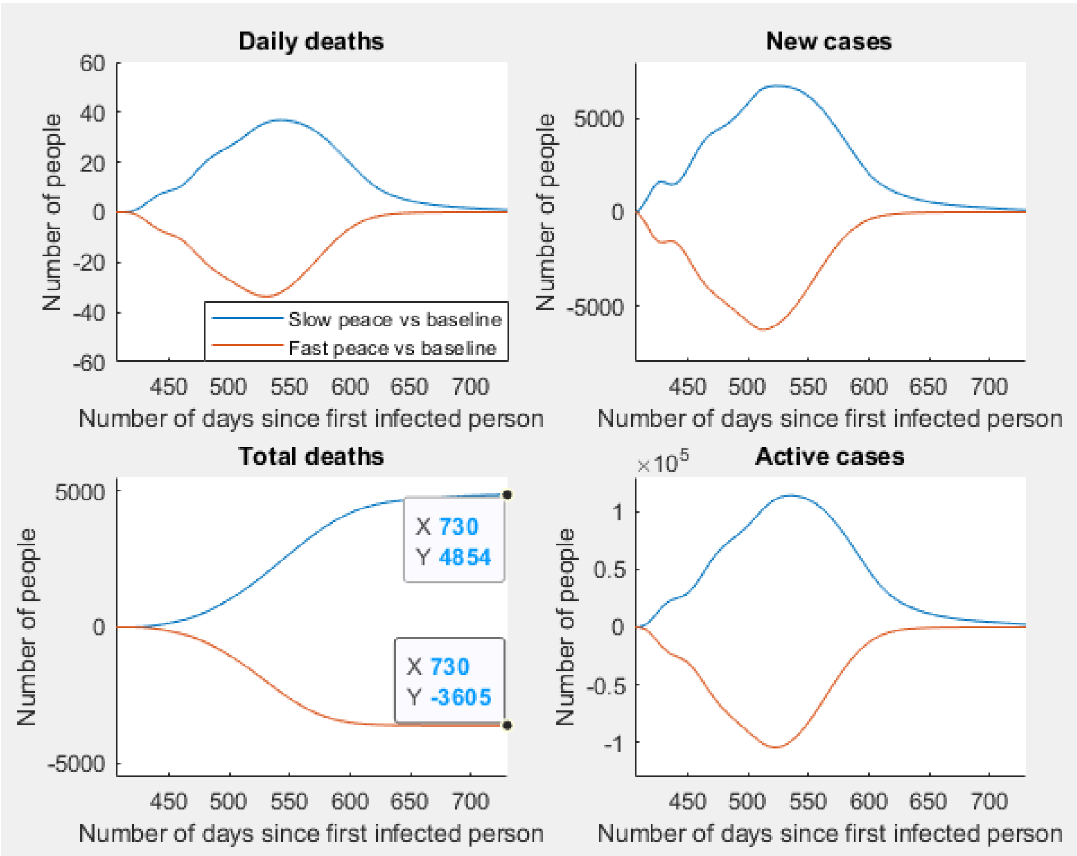
Summary of the effects for COVID-19 cases and deaths for different vaccination speeds.							
(number of people)	**Vaccination speed**	**End of the fourth wave Day 406 (10/03/21)**	**End of the fifth wave Day 450 (23/04/21)**	**Day 530 (12/07/2021)**	**Day 580 (31/08/2021)**	**Day 600 (20/09/2021)**	**Day 730 (28/01/2022)**
New cases	Slow	33,259	31,510	17,870	5,346	2,456.6	114.83
	Fast	33,259	26,741	5,483.5	210	26.84	0
Difference		**0**	**4,769**	**12,386.5**	**5,136**	**2,429.76**	**114.83**
Active cases	Slow	561,640	564,460	351,560	124,010	63,586	2,484.9
	Fast	561,640	503,800	135,600	9,219	1,622.9	0
Difference		**0**	**60,660**	**215,960**	**114,791**	**61,963**	**2,484.9**
Daily deaths	Slow	236.73	201.72	124.69	49,32	27.03	0.95
	Fast	236.73	184.44	55.24	5,2	1.06	0
Difference		**0**	**17.28**	**69.45**	**44,12**	**25.97**	**0.95**
Total deaths	Slow	74,301	83,314	96,185	100,460	101,200	101,980
	Fast	74,301	83,020	92,247	93,460	93,511	93,521
Difference		**0**	**294**	**3,938**	**7,000**	**7,689**	**8,459**

Source: Own elaboration.

In conclusion, both *figure 21* and *figure 22* along with *table 15* show and explain the importance of having a fast vaccination rate and the opportunity cost in terms of extra deaths that can be saved by doing so.

Regarding *figure 23*, the greatest difference between the two models in terms of COVID-19 cases and deaths take place during days 520-540 in the model. Therefore, I have chosen day 530 (12th of July 2021) to be represent it in the following *table 17* together with some other dates that I find key for analysing the evolution of the different scenarios. New cases, active cases, daily deaths, and total deaths increase up to somewhere around the 12th of July of 2021 and from that moment onwards they begin to decrease. This very same result was observable for the 1st of August 2021 (day 550) in the previous analysis when the immunity rate was altered instead of the vaccination rate.

Figure 23 COVID-19 incidence and deaths under different vaccination speeds.

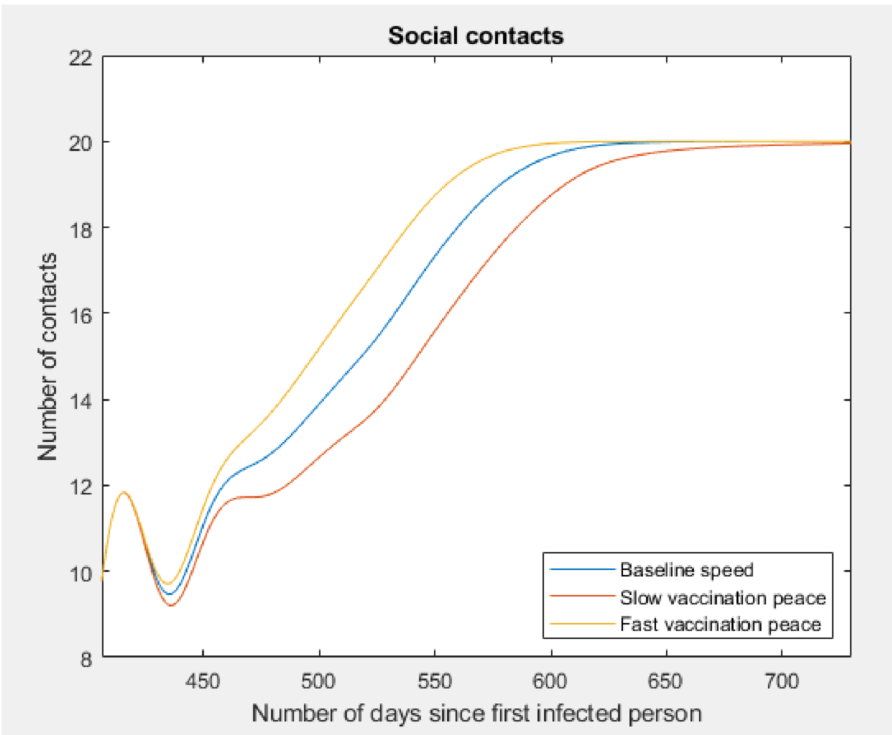


Source: Own elaboration.

On the other hand, it is also observable in *figure 22* how new cases for the case of low vaccination rate happen to be more sensible compared to a high vaccination peace. In *figure 23* it is observable how new cases experience a sixth wave after the fifth wave (which ends on day 450) that it does not take place under a fast vaccination speed (days 445-480 during mid-April until the end of May 2021).

Finally, *figure 24* shows the number of social contacts under this period of analysis (406-730). During this fifth wave, the number of social contacts is clearly decreased in the same proportion under the three scenarios. However, for the sixth and last wave in the model, it will take longer for the number of social contacts under a slow vaccination peace to begin to increase. It is not until around day 540 (21st of July 2021) when social contacts begin to increase at the same rhythm as under high vaccination peace. At that exact moment on the 21st of July (day 540), herd immunity is of 55.13% for a low-speed scenario compared to 71% for a high-speed scenario.

Figure 24 Social contacts under different vaccination speed scenarios.



Source: Own elaboration.

Consequently, as it can be observed, that by the 21st of July 2021, the positive effects of the vaccination are already strong and notorious (even if under low-speed scenario herd immunity of at least 60% is not yet reached) and the control of social contacts to hold the spread of the virus is no longer that necessary thanks to the vaccine. This allows the number of social contacts to increase at the same peace under both scenarios. Needless to say, that the same applies to the baseline scenario.

CONCLUSIONS

The baseline model in this paper has perfectly estimated Spanish official data on daily and total number of deaths. Overall, on the 28th of January 2022, after two years since the first infected individual in Spain, the model estimates more than 97.000 deaths. From the end of summer 2021 onwards (once herd immunity has already been achieved on the 31st of august 2021), daily deaths will nearly be zero together with daily infected and recovered individuals. At this moment, Spain will be able to attain pre pandemic situation in terms of the number of social contacts and restrictions will be relaxed. This has to do with the fact that the remaining susceptible individuals at the end of summer will represent a small share of the Spanish population (around 24%) mainly thanks to the share of those vaccinated and immune individuals (around 36% of the population).

However, the model does not perfectly match the number of active cases reported in the official data. This fact can be explained among other things by the delay of Spanish institutions in reporting daily new cases, or by the decision of individuals not to tell local authorities about having symptoms for the illness even if they are aware of it. Therefore, it can be said that the model is more accurate estimating the number of daily infected individuals as well as the total number of active cases given that model estimations perfectly fit the daily and total number of deaths for the period for which I have compared model results with Spanish official data (from the 29th of January 2020 up to the 10th of March 2021).

Over the period of analysis, the model has observed five waves for the pandemic caused by the COVID-19. However, the five waves have not had the same impact in the Spanish society. The first wave that lasted the same as the Spanish lockdown (more than three months from the 14th of March up to the 21st of Jun 2021) was the one that got more out of control in terms of daily deaths (highest peak). Nevertheless, if we consider the total number of deaths, the second and the third wave altogether were the most detrimental ones. This has to do with the fact that Spain didn´t carry out another lockdown that could have decreased daily infected individuals down to zero at any point during these two waves and also, with the fact that second and third waves have lasted much longer than the first wave without any clear timeout for control in between them (around seven months lengths since the 15th of July 2021).

As far as the vaccination is concerned, I have considered the first vaccinated individual in Spain took place on the 28th of January 2021, after one exact year since the first infected individual. For the first 41 days, the vaccination speed has been the equivalent to a 2.53% monthly vaccination share of the population and for that moment onwards it will be 7.5% of the population in order for Spain to attain herd immunity (set at 70% population share) at the end of summer 2021 (after a bit more than seven months since the first vaccinated individual). The main conclusions for the vaccination period under the baseline model are that with the arrival of the vaccine the number of daily recovered individuals has slowed down, and that daily herd immunity has been increasing at the same peace as the number of vaccinated and immune individuals. Besides, the vaccine weakens the contagion probability and empowers the testing rate which implies not needing to lower social contacts to hold the spread of the virus.

As far as the first part of the analysis is concerned, I have estimated the threats and opportunities in terms of the timing for reaching herd immunity when the efficacy rate of the vaccine captured by the immunity rate in the model is altered in 10% towards the baseline (baseline efficacy rate of 80%) from the 10th of March 2020 up to the end of the analysis on the 28th of January 2022. As it turned out, low efficacy rate of 70% delays reaching herd immunity by two months from the end of summer 2021 (baseline estimation) which in relative terms is doble what high efficacy rate of 90% can speed up herd immunity timing (that is less than one month). This has to do with the idea that time itself plays against herd immunity regardless of the efficacy rate. As time goes by, a higher proportion of the population will be vaccinated but will turn out not immune to the virus due the potential mutations of the virus COVID-19 that threaten the efficacy of the current vaccine.

The evolution of both models has shown that the greatest difference between the two scenarios regarding their impact in deaths and COVDI-19 cases (low and high efficacy rates) takes place around the 01st of August 2021 (day 550 in the model) when herd immunity reaches 60% for both cases. Sill, the negative effects of a low efficacy rate are greater than the positive effects of a high efficacy rate. In fact, a high efficacy rate has the potential to save 1,300 people while a low efficacy rate can trigger 1,700 more deaths by the 28th of January 2022 compared to the baseline model in which total deaths are around 97,000 individuals.

In the second and last part of the analysis, I have estimated the impact of having a 2.5% faster or slower monthly vaccination rate form the 10th of March 2021 onwards compared to the baseline monthly vaccination rate of 7.5% that is needed to reach herd immunity at the end of summer 2021. Results slow that a slow vaccination peace delays herd immunity by almost 4 months while a fast vaccination peace speeds it up by only a month and a half.

As far as the number of total deaths is concerned, a fast monthly vaccination peace of 10% of the Spanish population has the potential to save around 3,600 lives compared to the baseline while a monthly slow vaccination peace of 5% can bring about 4,800 deaths. Alike the results in the first part of the analysis it can be said that the negative impacts for the society due to a slow vaccination speed are greater in relative terms compared to the positive effects of a fast vaccination speed.

In conclusion, it is observable how the impact in the timing for reaching herd immunity is greater when changing the monthly vaccination speed (second part of the analysis) rather than when the efficacy rate of the vaccine is altered (in the first part of the analysis). Therefore, given that the efficacy rate of the vaccine cannot be controlled by vaccination policy makers, they should apply all their resources to attain the current desired monthly vaccination rate of 7.5% of the population in order to truly reach herd immunity by the end of summer (baseline model). Behold that this implies tripling the vaccination speed that the country has been following for the first 41 days and keeping it constant for almost six months since the 10th of March up to the 31st of August 2021. Sill, a monthly vaccination rate around 5%-10% ensures that by the 21st of July 2021 herd immunity of 60% is attained and that such a tight control of social contacts to hold back the spread of the virus is no longer that necessary thanks to having a big share of the population vaccinated and immune to the virus.

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